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# A Sequentially Compact Non-compact Quasi-pseudometric Space

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**Abstract.** We give an example of a sequentially compact non-compact quasi-pseudometric space, thus finding a negative answer to the problem posed by I. L. Reilly, P. V. Subrahmanyam and M. K. Vamanamurthy in [1].

## Example: Construction and Properties

The space considered in the following is due to R. Stoltenberg, see [2]. Let $R$ be the set of reals, let $J$ be the class of nonvoid finite or countable subsets of $R$. If $X, Y \in J$, we denote the usual distance between $X$ and $Y$ by $w(X, Y)$. It is simple to check the axioms of a quasi-pseudometric for the function on $J \times J$ defined by $d(X, Y) = \min(1, w(X - Y, Y))$ if $Y \subsetneqq X$, $d(X, X) = 0$ and $d(X, Y) = 1$ otherwise.

The following Lemma is used in Proposition 1.

**Lemma.** *Let $X$ be a non-empty subset of $R$ that does not contain the set of rationals $Q$, then $w(Q - X, X) = 0$.*

**Proposition 1.** *$(J, d)$ is sequentially compact.*

*Proof.* Let $(X_n : n \geqslant 1)$ be a sequence in $J$. We may suppose that for each $n \geqslant 1$, there is $m > n$ such that $X_m$ does not contain $Q$ (otherwise the sequence would converge to $X = \bigcup \{X_i : i \geqslant 1\}$). A subsequence $(X_{n_k} : k \geqslant 1)$ may be constructed inductively such that, for all $k$, $X_{n_k}$ does not contain $Q$. It is easily seen that $(X_{n_k} : k \geqslant 1)$ converges to $X = Q \cup (\bigcup \{X_i : i \geqslant 1\})$, which is in $J$.

**Proposition 2.** *$(J, d)$ is not compact.*

*Proof.* If $X \in J$, let $P(X)$ be the class of non-empty subsets of $X$. Now for every $Z \in P(X)$, the open ball $B(Z, \frac{1}{2}) \subset P(Z) \subset P(X)$. Thus,

$\mathscr{U} = \{P(X): X \in J\}$ is an open cover of $J$ which obviously does not admit a finite subcover.

## References

[1] REILLY, I. L., SUBRAHMANYAM, P. V., VAMANAMURTHY, M. K.: Cauchy sequences in quasi-pseudo-metric spaces. Mh. Math. **93**, 127—140 (1982).

[2] STOLTENBERG, R.: Some properties of quasi-uniform spaces. Proc. London Math. Soc. (3) **17**, 226—240 (1967).

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