Werk

Titel: A Course in Density Estimation - Devroye, L.

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finite samples and vectors with a finite number of components, (T) concentrates on illustrations and examples of applications to stochastic processes and random functions.

We summarize the contents of (T). After a short introduction to the measure-theoretic tools the authors stress in Chapter II the integral representation of (discrimination) information, characterize sufficiency, and prove the minimum information theorem. Chapter III deals with limiting properties of the information. In Chapter IV on product measure spaces the Ornstein-Uhlenbeck process, the Wiener process and general Gaussian Markov processes are introduced. Moreover, in this chapter log Likelihood ratios associated with stochastic processes are computed by an application of the minimum information estimates. Chapter V contains most valuable knowledge on linear discrimination functionals. They are used in the information-theoretic analysis of various stochastic processes. There is a concluding chapter on canonical correlation.

The "Topics", although selected somewhat narrowly from the authors' special field of interest during the last twenty years, provide useful material for probabilists and statisticians working in Gaussian processes or statistical decision theory respectively. One must not forget, however, that information discrimination has also been analysed via divergences, deficiencies, insufficiencies, Hellinger transforms and other "distances" between measures. In these approaches the basic motivation is the idea that there exists no functional which carries in a satisfactory manner the entire information contained in an experiment. Consequently the study of comparisons of experiments in the sense of information gain or loss makes real sense. Supports of the "total containment" point of view (like some of the Bayesians) have had little success to enhance significantly asymptotic statistical decision theory, a goal which has been more efficiently achieved by those following the "comparison" point of view. Solomon Kullback's contributions, in particular the results presented in (T), are excellent sources to look into if one wants to understand this critical alternative.

Tübingen *H. Heyer*

Devroye L: *A course in density estimation*. Birkhäuser 1987, XIX + 183 pp, DM 98,–.

Once the reader has accepted the author's opinion that the L_1 distance of a density estimate from the target density is the only proper criterion for judging its goodness, the booklet by Luc Devroye offers a short but well-rounded course in density estimation based on independent and identically distributed observations.

There is clearly a great overlap with the earlier book by Devroye and Györfi (1985), "Nonparametric Density Estimation. The L_1 View", Wiley, on the same subject. But while that one is a research monograph, the present text is organized like lecture notes for graduate students. The author seeks to explain and to provide more didactic proofs and, for this reason, focuses on the kernel density estimate.

Chapter I is a toolbox of inequalities between the L_1 distance of densities and other popular distances such as the Hellinger or the Kullback-Leibler distance.

In Chapter II a list of popular density estimates is presented (histogram, kernel estimate, nearest neighbor estimate) as well as a collection of basic material for further purposes (characteristic functions, Scheffé's theorem. Lebesgue density theorem).

Chapter III is devoted to the proof of a quite deep result by the author which characterizes L_1 consistency of the kernel density estimate.

In Chapter IV the author shows that the expected L_1 distance of the kernel estimate is insensitive to slight deviations of the shape of the underlying density. This attribute is called robustness of the kernel density estimate, but it is rather a continuity property of the corresponding L_1 distance.

In Chapter V lower bounds for the minimax risk of an arbitrary density estimate over different classes of underlying densities are derived. Unfortunately, as the author points out, there does not seem to exist up to now a systematic approach of how to find minimax-optimal estimators which attain these bounds.

An attempt is made in Chapter VI, where it is shown that minimum distance estimates are minimax optimal up to a constant, if the underlying class of densities is a particular one which is quite small. Though mathematically appealing, the practical importance of minimum distance estimates is limited due to their computational inefficiency.

In the long Chapter VII upper bounds for the L_1 distance of the kernel estimate uniformly over a given class of densities as well as for a single density are established. These results are completely analogous to the well-known results on the L_2 distance of the kernel density estimate, and it becomes apparent from this chapter why some authors argue that there is hardly any difference between density estimates optimized for L_1 or L_2 distance.

Chapter VIII is a case study on monotone density estimation in which various density estimates are studied.

The main result of Chapter IX is an exponential probability bound for the L_1 distance J_n of the kernel density estimate, centered by its expectation $E(J_n)$. This exponential bound entails a list of consequences, among which here is a law of large numbers for J_n, i.e. the a.s. convergence of $J_n/E(J_n)$ to one. A toolbox of inequalities concludes this section.

The author studies in his booklet various interesting aspects of density estimation in detail and gives in addition references to relevant problems which are not covered within the text. The clear presentation together with the numerous exercises, the many figures and, last but not least, the author's enthusiasm for the subject which can be read between the lines make it a suitable basis for a first course on density estimation as well as an entry into the research monograph with L. Györfi mentioned above.

Siegen *M. Falk*