Werk

Titel: Introduction.

Jahr: 1991

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Invent. math. 104, 545–580 (1991)

Inventiones mathematicae
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The dynamics of complex polynomials and automorphisms of the shift

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Oblatum 6-XII-1989 & 13-IX-1990

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Introduction

The focus of this article is the interplay between two distinct areas of dynamical systems. The first is the study of iteration of polynomials in one complex variable; the second is the study of automorphisms of the one-sided shift.

The most conspicuous feature of a complex polynomial is its *filled-in Julia set*; this is the set of points whose orbits stay bounded under iteration. The filled-in Julia set of a quadratic polynomial is one of two types: it is either connected in which case the unique critical point belongs to the filled-in Julia set, or it is a Cantor set and the critical point escapes to ∞. In higher degrees the situation is more complicated.

We consider the space X_d of monic, centered polynomials of degree d; these are polynomials of the form $z^d + a_{d-2} z^{d-2} + \ldots + a_0$. Every polynomial of degree

* Research supported in part by the National Science Foundation grant # DMS 88-01277
** Research supported in part by the National Science Foundation grants # DMS 870105 and # RII-8903049

d is affine conjugate to one that is monic and centered (by a conjugacy that need not be unique). The *connectedness locus* is the subset of X_d consisting of polynomials whose filled-in Julia set contains all the critical points. The other extreme is the subset, S_d, called the shift locus. This set contains the *shift-like* polynomials; these are polynomials whose filled-in Julia set contains no critical points. In this case the filled-in Julia set is a Cantor set. If $d>2$ there are many polynomials that are in neither the shift locus nor in the connectedness locus. In fact, there are polynomials whose Julia set is a Cantor set but which are *not* shift-like [Br], [B2].

This paper is motivated by what, at first, seems to be a question in combinatorial dynamical systems – the structure of the group automorphisms of the one-sided d-shift. Recall that the sequence space, Σ_d, is the space of semi-infinite sequences with entries from an alphabet of d symbols and that the shift map acts on sequences by dropping the first entry of each sequence. The *automorphism group* Aut_d consists of homeomorphisms of Σ_d that commute with the shift. The groups Aut_d were originally studied by Hedlund [H] in the 1960's; he proved the important result that Aut_2 consists only of the identity and the symbol interchange. More recently, Boyle, Franks, and Kitchens [BFK] showed, by contrast, that when $d>2$, Aut_d is infinitely generated with a rich algebraic structure. Furthermore, Ashley [Ash] exhibited an efficient set of generators called *minimal marker automorphisms.*

Our goal in this paper is to show that there is a surprising relationship between the topology of the shift locus and Aut_d. In particular, we present a geometric method for realizing automorphisms of the d-shift via loops in the shift locus.

The bridge between shift-like polynomials and automorphisms of the shift relies on the following fact: if P is a shift-like polynomial of degree d, its dynamics, when restricted to its Julia set, are conjugate to the one-sided shift on d symbols. Moreover, since the polynomial restricted to the Julia set is hyperbolic, the Julia set varies continuously as we change the polynomial within the shift locus. Thus, a closed loop in the shift locus induces an automorphism of the shift in the natural way: we simply follow points in the corresponding Julia sets as we traverse the loop and the induced map on the Cantor set is an automorphism of the d-shift. We call this automorphism the monodromy induced by the loop. For example, if we look at the monodromy induced by following a loop which winds once around the Mandelbrot set, then we find that the induced automorphism is the unique non-trivial element of Aut_2. In general, we have a representation:

$$\Theta_d\colon\ \pi_1(S_d)\to \mathrm{Aut}_d.$$

Our main result is:

Theorem *The map Θ_d is a surjection.*

Our methods involve two similar constructions, both of which relate the structure of the dynamical plane to that of the parameter space. The first construction involves "spinning the lowest critical value" around its level curve (in the dynamical plane) of the rate of escape potential. This generates loops in the shift locus which essentially produce the minimal marker automorphisms. The second construction deforms the polynomial through different levels of the potential.