$$f = \text{constant}) \cdot \prod_{a \in (\mathbf{Z}/p\mathbf{Z})^*} g_{0,a}^{\chi(a)},$$

where $\chi(a) = (a/p)$ is the Legendre symbol. Here I am using the obvious modification of Kubert-Lang's notation for Siegel functions, i.e., (a, b) in $\mathbf{Z}/p\mathbf{Z} \times \mathbf{Z}/p\mathbf{Z}$ rather than in $1/p\,\mathbf{Z}/\mathbf{Z} \times 1/p\,\mathbf{Z}/\mathbf{Z}$ [39].

To compute f^w we use a result of Kubert and Lang (§ 4 of [39]) which applied to our case gives:

$$f^w = (\text{constant}) \cdot \prod_{\substack{a \in (\mathbf{Z}/p\mathbf{Z})^* \\ b \in \mathbf{Z}/p\mathbf{Z}}} g_{a^{-1},b}^{\chi(a)}.$$

Now let $\equiv$ denote "mod squares":

$$f \cdot f^w = \prod_{a \in (\mathbf{Z}/p\mathbf{Z})^*} g_{0,a} \cdot \prod_{\substack{a \in (\mathbf{Z}/p\mathbf{Z})^* \\ b \in (\mathbf{Z}/p\mathbf{Z})}} g_{a^{-1},b} = \prod_{\substack{(a,b) \in \mathbf{Z}/p\mathbf{Z} \times \mathbf{Z}/p\mathbf{Z} \\ (a,b) \neq (0.0)}} g_{a,b}$$

and the latter product of Siegel functions is one.

3. Page 100, line 7 bot.. The formula for the action of T_l on $X_1(N)$ quoted here is incorrect. The correct formula is, of course, well known (cf., e.g., [67], § 2), and the argument on the next page in which the formula is used is unaffected by the correction.

(B.M.)

II. Erata for [67]

Hida has pointed out that the formulas for U_p' in Theorem 5.3 are not correct as they stand. They should state:

On $\mathrm{Pic}^0(\mathscr{C}_\infty^{(N)})$, $U_p' = \langle n_p^{-1} \rangle (\mathrm{Frob}_p + \sum_{\zeta \neq 1} W_\zeta)$.

On $\mathrm{Pic}^0(\mathscr{C}_0^{(N)})$, $U_p' = \mathrm{Ver}_p$.

The proof is correct except for the formula giving the conjugation of Frob_p and Ver_p by W. They should read:

$$W^{-1}(\mathrm{Frob}_p) W = \mathrm{Frob}_p \cdot \langle n_p^{-1} \rangle, \quad W^{-1}(\mathrm{Ver}_p) W = \mathrm{Ver}_p \cdot \langle n_p \rangle.$$

(A.W.)

Chapter 0. Notation and preliminaries

If K is a field, $\bar{K}$ denotes an algebraic closure. If K is a local or global field, $\mathcal{O}(K)$ is its ring of integers.

If Y is a scheme over a base S and $T \to S$ any base change, $Y_{/T}$ denotes the pullback of Y to T. If $T = \mathrm{Spec}\, A$, we may also denote this scheme by $Y_{/A}$. By $Y(T)$ we mean the T-rational points of the S-scheme Y, and again, if $T = \mathrm{Spec}\, A$, we may also denote this set by $Y(A)$.

If $A_{/T}$ is a group scheme and N an integer, $A[N]_{/T}$ is the kernel of multiplication by N in A, viewed as group scheme over T.

A finite flat group scheme G over S is said to be of *multiplicative-type* if its Cartier dual is étale; it is called *ordinary* for every geometric point s of S, the fibre G_s is a product of a multiplicative-type group scheme and an étale group scheme.

Proposition. *Let K be a finite extension of $\mathbf{Q}_p$ of ramification index not divisible by $p-1$. Let φ: $G_1 \to G_2$ be a morphism of ordinary finite flat group schemes over $\mathcal{O}(K)$ such that the induced map*

$$\varphi(\bar{K}): \quad G_1(\bar{K}) \to G_2(\bar{K})$$

is injective. Then if k is the residue field of $\mathcal{O}(K)$, the induced mapping

$$\varphi(\bar{k}): \quad G_1(\bar{k}) \to G_2(\bar{k})$$

is also injective.

Remark: Although this is all we shall use, much more is true: Under the same hypotheses on K, and G_1, G_2, any injection $\varphi_{/K}$: $G_{1_{/K}} \hookrightarrow G_{2_{/K}}$ extends to a closed immersion of G_1 to G_2.

If we *drop* the hypotheses that G_1 and G_2 are ordinary, but require that the ramification index of K be *less than* $p-1$, these results remain true by the work of Oort-Tate ([52]; for groups of order p); of Raynaud ([55]; for groups of type $(p, p, \ldots, p)$; and of Fontaine ([20]; for arbitrary groups).

Before we give the proof of this proposition, recall the following consequences of the theory of Oort-Tate [52]:

Any finite flat group scheme G of order p over $\mathcal{O}(k)$ is isomorphic to a group scheme of the form $G_{a,b}$ for a, b elements of $\mathcal{O}(K)$ such that $a \cdot b = w_p \cdot 1$ where w_p is equal to p times a unit. [52], §2; especially Theorem 2). The pair (a, b) is uniquely determined by the isomorphism class of G up to multiplication by $(p-1)$-st powers of units in $\mathcal{O}(K)$, i.e. $G_{a,b} \underset{\mathcal{O}(k)}{\cong} G_{a',b'}$ if and only if $a' = u^{p-1} \cdot a$, $b = u^{p-1} \cdot b'$ for some $u \in \mathcal{O}(K)^*$.

The group schemes $G_{a,b}$ and $G_{a',b'}$ are isomorphic over K (i.e. they have isomorphic Galois representations) if and only if

$$a' = r^{p-1} \cdot a; \quad b = r^{p-1} \cdot b'$$

for some $r \in K^*$.

The group scheme $G_{a,b}$ is of multiplicative type if and only if b is a unit in $\mathcal{O}(K)$; it is étale if and only if a is a unit.

Lemma 1. *Let G, G' be finite flat group scheme of order p over $\mathcal{O}(K)$, where G is étale and G' is of multiplicative type. Suppose that the ramification index of K over $\mathbf{Q}_p$ is not divisible by $p-1$. Then there are no nontrivial homomorphisms over K from G to G'.*

Proof. Let $G = G_{a,b}$; $G' = G_{a',b'}$ where a and b' are units in $\mathcal{O}(K)$. Let v be the valuation of K, normalized so that if π is a uniformizer then $v(\pi) = 1$.

By our hypothesis, $v(p) \not\equiv 0 \bmod (p-1)$. But if there were a nontrivial homomorphism (hence isomorphism) from $G_{/K}$ to $G'_{/K}$ then $b = r^{p-1} \cdot b'$ for some $r \in K^*$ and $v(p) = v(b) = (p-1) \cdot v(r) + v(b') = (p-1) \cdot v(r)$ yields a contradiction.

We now return to the proposition.

Replacing K by a finite unramified extension, we may suppose that the étale quotients $G_1^{\text{ét}}$, $G_2^{\text{ét}}$ are constant group schemes, and the connected components G_1^0, G_2^0 are the Cartier duals of constant group schemes. By our hypotheses on K, there are no nontrivial mappings between constant group schemes over K and Cartier duals of constant group schemes (in either direction). It follows that the inverse image of $G_{2/K}^0$ under $\varphi_{/K}$ is $G_{1/K}^0$ and consequently induces an injection of étale quotients

$$\varphi^{\text{ét}}: \quad G_1^{\text{ét}} \hookrightarrow G_2^{\text{ét}},$$

proving the proposition.

In this paper we shall often be given p-divisible group schemes $\Gamma_{/\mathbf{Q}_p}$ such that for some finite field extension K of $\mathbf{Q}_p$, the "base change" $\Gamma_{/K}$ is isomorphic to the generic fibre of a p-division group scheme over $\mathcal{O}(K)$. By a theorem of Tate, this group scheme over $\mathcal{O}(K)$ is uniquely determined (up to a canonical isomorphism) by $\Gamma_{/K}$. We shall call it $\Gamma_{/\mathcal{O}(K)}$ and refer to it as the *prolongation* of $\Gamma_{/K}$ *over the base* $\mathcal{O}(K)$. By the uniqueness theorem, one has that prolongations "commute with base change".

Given a *finite flat* group scheme G over K it is not necessarily the case that it admits at most one *prolongation* to a finite flat group scheme over $\mathcal{O}(K)$.

Corollary to the Proposition. *Let K be as in the proposition and let $i_{/K}$: $\Gamma_{/K} \to \Gamma'_{/K}$ be an injection of p-divisible group schemes over K. Suppose that $\Gamma_{/K}$ and $\Gamma'_{/K}$ have ordinary prolongations over $\mathcal{O}(K)$. Then the unique homomorphism*

$$i: \Gamma_{/\mathcal{O}(K)} \to \Gamma'_{\mathcal{O}(K)}$$

which extends $i_{/K}$ ([64] Theorem 4) induces an injection on $\bar{k}$-valued points:

$$i(\bar{k}): \quad \Gamma(\bar{k}) \hookrightarrow \Gamma'(\bar{k}).$$

Remark. The morphism i is, in fact, a closed immersion.

For use in Chap. 4 we include the following result.

Lemma. *Let K be a finite extension field of $\mathbf{Q}_p$ and $\mathcal{O} = \mathcal{O}(K)$.*

Let $\mathcal{N}_{/\mathcal{O}}$ be the Néron model of an abelian scheme $A_{/K}$. Suppose $\mathcal{N}_{/\mathcal{O}}$ has semi-stable reduction.

Let $G_{/K} \subseteq A_{/K}$ be a finite subgroup scheme, and $G_{/\mathcal{O}}$ the Zariski-closure of $G_{/K}$ in $\mathcal{N}_{/\mathcal{O}}$. Suppose that the splitting field of the action of Gal$(\bar{K}/K)$ *on* $G(\bar{K})$ *is an unramified extension of K.*

Then $G_{/\mathcal{O}}$ is a finite flate (étale) group scheme.

Proof. The Néron model commutes with unramified base change; schematic closure commutes with flat base change. Therefore we may suppose the Galois action on $G_{/K}$ trivial. Let $\mathcal{G}_{/\mathcal{O}}$ denote the constant group scheme $G(K)$ over $\mathcal{O}$. Since $\mathcal{N}(\mathcal{O}) \to \mathcal{N}(K)$ is a bijection, we have a morphism u: $\mathcal{G}_{/\mathcal{O}} \to \mathcal{N}_{/\mathcal{O}}$ such that $u_{/K}$ is an isomorphism of $\mathcal{G}_{/K}$ onto $G_{/K}$. Therefore u factors through the schematic closure $G_{/\mathcal{O}}$. Since $\mathcal{G}_{/\mathcal{O}}$ is finite, so is $G_{/\mathcal{O}}$.