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Kontakt/Contact

Digizeitschriften e.V.
SUB Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen

info@digizeitschriften.de

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Regularity of the Singular Sets of Two-Dimensional Area-Minimizing Flat Chains Modulo 3 in R^3

Jean E. Taylor (New Brunswick)

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Introduction

Determining the existence and structure of surfaces of minimum area having a given boundary is a problem of long-standing interest. The success of geometric measure theoretic methods in recent years in showing the existence and regularity almost everywhere of solutions to a variety of different formulations of the problem of least area (the class of problems frequently collectively called Plateau's Problem) has now focused increased attention on the problem of determining the structure of the non-regular, or singular, points of these solutions. The results of this thesis are a contribution to the study of the lower dimensional regularity of certain of these singular sets.

The class of surfaces we consider here, namely rectifiable flat chains modulo 3 whose boundaries are also rectifiable flat chains modulo 3, can be defined in a number of ways [5, 4.2.26; 7]. They are characterized by the property that both they and their boundaries simultaneously agree, except in a set of arbitrarily small two and one-dimensional area respectively, with the images of C^1 singular chains with coefficients in the integers mod 3, the particular chains depending on the degree of approximation desired. One dimensional singularities can arise in these surfaces because of cancellation modulo 3. For instance, consider the surface Y consisting of three half disks with common diameter and at angles of 120° to each other (see Fig. 1; in spherical coordinates

$$Y=\{(r,\theta,\varphi)\colon r\leqq 1, \theta=0 \text{ or } 2\pi/3 \text{ or } 4\pi/3\}).$$

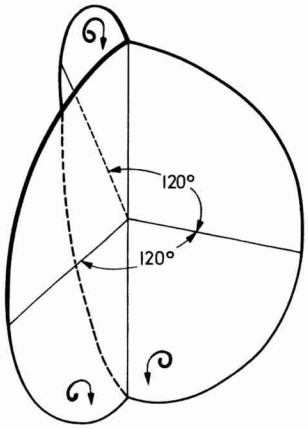

Fig. 1. The flat chain modulo 3 Y

This Y is the support of a rectifiable flat chain modulo 3 with boundary support consisting of three half circles; indeed, Y (suitably oriented) is the area minimizing flat chain modulo 3 having this boundary (correspondingly oriented) and it represents the soap film which physically forms on a wire frame consisting of three such half circles.

The compactness theorem for rectifiable flat chains modulo 3 [5, 4.2.26, 4.2.17$^{\nu}$] assures that for any one-dimensional rectifiable cycle modulo 3 S in $\mathbf{R}^3$, there exists a two dimensional area-minimizing rectifiable flat chain modulo 3 T in $\mathbf{R}^3$ such that $\partial T = S$; except for a locally compact interior singular set $\sigma(T)$ of zero two dimensional Hausdorff area, spt $T \sim S$ has been proved to be a two-dimensional real analytic submanifold of $\mathbf{R}^3$ having zero mean curvature at each of its points [5, 5.3.19; 8, 6.7.6]. We prove here that the interior singular set $\sigma(T)$ is locally a one-dimensional Hölder-continuously differentiable submanifold of $\mathbf{R}^3$ and give the complete local structure of the class of surfaces described above. In particular we note that there are no a priori symmetry or other assumptions about T or S. (The structure of spt T along spt S is still only partially understood – see [1].)

Initially, all that one knows is that the singular set is a compact set of zero two-dimensional Hausdorff measure [5, 5.3.9]. However, using well-known theorems in geometric measure theory and combinatorial arguments specific to flat chains modulo 3, we show (Theorem 1.5(3)) that the entire singular set consists of points each of whose tangent cones is equal to the extension of Y up to a rotation. (In general, the uniqueness of tangent cones at singular points of area minimizing surfaces is still an open question.) Following this, and central to this thesis, is the proof