take ε such that $\|\underline{u}'\| < |M(s_1)|$ and $\underline{u}(t)$ is a strict upper solution of $(1)_s$, (2). $-|M(s_1)|(t-\eta)$ is a strict lower solution of $(1)_s$, (2). According to Lemma 5 for $s \in (\mathrm{s}, s_1]$ we have

$$d_L(L+N_s, \Omega_\varepsilon) = \pm 1 \pmod 2. \tag{14}$$

From the additivity property of the degree it follows that

$$d_L(L+N_s, \Omega_1 - \overline{\Omega}_\varepsilon) = \pm 1 \pmod 2 \tag{15}$$

for $s \in (\mathrm{s}, s_1]$. Relations (14), (15) imply the existence of a solution of the BVP $(1)_s$, (2) in Ω_ε and in $\Omega_1 - \overline{\Omega}_\varepsilon$. Since s is arbitrary in (s_0, s_1), the BVP $(1)_s$, (2) has at least two solutions for $s \in (s_0, s_1]$.

Now we prove that $(1)_s$, (2) has a solution for $s = s_0$. Let us take a sequence $\{s_n\}_{n=1}^{\infty}$, where $s_n \in (s_0, s_1]$, $n \in N$, $\lim\limits_{n\to\infty} s_n = s_0$. We know that for any s_n $(1)_s$, (2) has a solution u_n satisfying $\|u_n\| < |M(s_1)|$, $\|u_n'\| < |M(s_1)|$, and according to Lemma 1 we get $\|u_n''\| < \varrho$ for ϱ large enough. Since u_n is a solution of $(1)_{s_n}$, (2) the sequence $\{u_n'''\}_{n=1}^{\infty}$ is bounded in $C^0(0,1)$. By the Arzela-Ascoli lemma we can suppose that $\{u_n\}_{n=1}^{\infty}$ converges in $C^2(0,1)$ to a solution of $(1)_s$, (2). Theorem 5 is proved. □

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