Werk

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If we now introduce the abbreviations

$$k_\delta^{(n)}(x) = \frac{1}{\delta} k^{(n)}\left(\frac{x}{\delta}\right) \quad \text{and} \quad k_\delta(x) = \frac{1}{\delta} k\left(\frac{x}{\delta}\right),$$

and use the notation $(u * v)(x)$ to denote $\int u(x-\xi)\, v(\xi)\, d\xi = \int u(\xi)\, v(x-\xi)\, d\xi$, the convolution of u and v evaluated at x, we may write

$$\begin{aligned}|(k_\delta * g)(x) - \gamma\, g(x)| \leqq |(k_\delta^{(n)} * g)(x) - \gamma_n\, g(x)| + \\ + |([k_\delta - k_\delta^{(n)}] * g)(x)| + |(\gamma_n - \gamma)\, g(x)|\,.\end{aligned}$$

For the middle term we have the estimate

$$|([k_\delta - k_\delta^{(n)}] * g)(x)| \leqq \|k_\delta - k_\delta^{(n)}\|_1\, \|g\|_\infty = \|k - k^{(n)}\|_1\, \|g\|_\infty\,;$$

hence

$$\begin{aligned}|(k_\delta * g)(x) - \gamma\, g(x)| \leqq \\ \leqq |(k_\delta^{(n)} * g)(x) - \gamma_n\, g(x)| + [\|k - k^{(n)}\|_1 + |\gamma_n - \gamma|]\, \|g\|_\infty\,.\end{aligned}$$

Since each of the $k^{(n)}$'s is a bounded measurable function vanishing outside of a finite interval, by what has already been proven, the first term on the right of the preceding estimate tends to zero as $\delta \downarrow 0$, for x in the Lebesgue set of g. Thus, on the Lebesgue set of g

$$\overline{\lim_{\delta \downarrow 0}}\, |(k_\delta * g)(x) - \gamma\, g(x)| \leqq [\|k - k^{(n)}\|_1 + |\gamma_n - \gamma|]\, \|g\|_\infty\,;$$

so that in view of (3) and (4), the desired result (2) then follows by letting $n \to \infty$.

References

[1] *N. Dunford* and *J. T. Schwartz*, Linear operators, Part I, Interscience, New York, 1958.
[2] *N. Wiener*, Tauberian theorems, Annals of Math. *33* (1932), 1—100.

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