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Обозначим еще через $r(T_p)$ спектральный радиус T_p. В части 5 прежде всего отмечается, что для возвратной цепи $r(T_p) = \|T_p\|_{p,\mu} = 1$. Потом построен ряд примеров, показывающих, что для невозвратной цепи может случиться $r(T_2) < 1$, $\|T_2\|_{2,\mu} < 1$ для μ инвариантной и для μ неинвариантной; может случиться $r(T_2) = \|T_2\|_{2,\mu} = 1$ для μ инвариантной и для μ неинвариантной; может случиться $r(T_2) < 1$, $\|T_2\|_{2,\mu} = 1$ (для μ неинвариантной).

Часть 6 показывает пример цепи, для которой число 0 лежит в точечном спектре T_1, в непрерывном спектре T_2 и в резидуальном спектре T.

В части 7 построена приводимая цепь, для которой спектр T равен целому единичному кругу.

В сущности статья содержит некоторые дополнения и примеры к предыдущей работе [13].

Summary

SOME THEOREMS AND EXAMPLES IN THE THEORY OF OPERATORS IN DENUMERABLE MARKOV CHAINS

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Let us consider a Markov chain in discrete time, with a denumerable state space and stationary transition probabilities p_{jk}; let μ be its sub-invariant measure (see [10], [11]).

Denote by $l_\infty(\varepsilon)$ the Banach space of all bounded complex functions f on the state space with the norm $\|f\|_{\infty,\varepsilon} = \sup_k |f_k|$; and by $l_p(\mu)$ (for $1 \leqq p < \infty$) the space of functions f for which the norm $\|f\|_{p,\mu} = (\sum_k |f_k|^p \mu_k)^{1/p}$ is finite.

Define the operator T in $l_\infty(\varepsilon)$ so that the value of Tf at the point j is $(Tf)_j = \sum_k f_k p_{jk}$. The same formula defines the operator T_p (for $1 \leqq p < \infty$) for functions f from $l_p(\mu)$. The norms of these operators are $\|T\|_{\infty,\varepsilon} = 1$, $\|T_p\|_{p,\mu} \leqq 1$ (see [11]).

In section 2, the form of operators adjoint to T and T_p is shown; and the connection with the operator used by Kendall [10] is examined.

Section 3 contains several theorems stating that for an irreducible recurrent chain and for f from certain classes of functions, the inequality $Tf \leqq f$ implies $Tf = f$, and similarly for T_p and for the adjoint operators.

Section 4 proves that for an irreducible recurrent chain, the equation $Tf = f$ has an essentially unique solution f, and again similarly for T_p and for the adjoint operators. Sections 3 and 4 systematically complete some special known results from [2], [5], [13].

Denote by $r(T_p)$ the spectral radius of T_p. In section 5, it is first shown that for a recurrent chain $r(T_p) = \|T_p\|_{p,\mu} = 1$. Then a series of examples is constructed showing that for a transient chain different cases can happen: the case $r(T_2) < 1$, $\|T_2\|_{2,\mu} < 1$ for μ invariant and for μ non-invariant; the case $r(T_2) = \|T_2\|_{2,\mu} = 1$ for μ invariant and for μ non-invariant; the case $r(T_2) < 1$, $\|T_2\|_{2,\mu} = 1$ (for μ non-invariant).

Section 6 contains an example of a chain for which the number 0 lies in the point spectrum of T_1, in the continuous spectrum of T_2, and in the residual spectrum of T.

In section 7 a reducible chain is constructed for which the spectrum of T is equal to the whole unit circle.

The paper consists of some supplements and examples to the previous paper [13].