Summary

ON A CLASS OF l-GROUPS

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Let G be an l-group (see [2]). We denote by $[G]$ the lattice of all l-ideals of G. If $A \subset G^+$, we put $K'(A) = E\ (x\colon x \cap y = 0$ for each $x \in A)$, $K(A) = K'(K'(A))$. We consider l-groups satisfying the following condition:

(P) If $A \subset G^+$, $x \in G^+$, there exist elements $y \in K(A)$, $z \in K'(A)$ such that $x \leqq y \cup z$.

G. Birkhoff proved the theorem ([1], § 18, "main structure theorem", theorem 33):

(B) *Let G be a commutative l-group, whose lattice $[G]$ has finite length. Then either G is a direct product or G contains a unique maximal proper l-ideal.*

Commutative l-groups need not satisfy the condition (P) and l-groups with property (P) need not be commutative. Nevertheless, the structure of l-groups with the property (P) is analoguous to the structure of commutative l-groups.

In theorems 1—3 we suppose that G has the property (P).

Theorem 1. *Either G is an ordered (= linearly ordered) group or G is a direct product.*

Theorem 2. *If $[G]$ satisfies the descending chain condition, then G is a direct product of a finite number of ordered groups.*

Theorem 3. *If $[G]$ satisfies the ascending chain condition, then either G is a direct product or G contains a unique maximal proper l-ideal.*

We will denote by $B(0)$ the intersection of all l-subgroups $G_i \subset G$ such that $x \in G$, $x \parallel 0 \Rightarrow x \in G_i$. (The symbol $x \parallel y$ means that the elements x, y are incomparable.) It is proved that $B(0)$ si an l-ideal in G; set $G/B(0) = \bar{G}$.

A generalization of the theorem 3 is:

Theorem 4. *If $B(0)$ satisfied the condition* (P) *and if $[\bar{G}]$ satisfies the ascending chain condition, then either G is a direct product or G contains a unique maximal proper l-ideal.*

A generalization of the theorem (B) is

Theorem 4'. *If the l-group $B(0)$ is commutative and if the lattice $[\bar{G}]$ has finite length, then either G is a direct product or G contains a unique maximal proper l-ideal.*