Werk

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а $\mathfrak{M}(\boldsymbol{u}_1, X_1)$ обозначает множество всех действительных чисел t, для которых $X_1 + t\boldsymbol{u}_1 \in \mathfrak{M}$. Если точке X_2 соответствует тот же базис, как и точке X_1, то положим $X_3 = X_2 + t_2\boldsymbol{u}_2$, где t_2 определяется условием

$$f(X_2 + t_2\boldsymbol{u}_2) = \min_{t \in \mathfrak{M}(u_2 X_2)} f(X_2 + t\boldsymbol{u}_2) .$$

Если же точке X_2 не соответствует тот же базис, что точке X_1, то для X_2 повторяем процесс, примененный к X_1, с первым вектором соответствующего точке X_2 базиса.

Предположим, что мы уже построили точку X_n. Пусть $\mathbf{v}_1^{(n-1)}, \ldots, \mathbf{v}_k^{(n-1)}$ — векторы соответствующего точке X_{n-1} базиса и пусть $X = X_{n-1} + t_{n-1}\mathbf{v}_r^{(n-1)}$ $(1 \leqq r \leqq k)$; если точке X_n соответствует тот же басис, как и точке X_{n-1}, положим $X_{n+1} = X_n + t_n\mathbf{v}_{\bar{r}}^{(n-1)}$, где $\bar{r} \equiv r + 1 \pmod{k}$, $1 \leqq \bar{r} \leqq k$ и t_n определяется условием

$$f(X_n + t_n\mathbf{v}_{\bar{r}}^{(n-1)}) = \min_{t \in \mathfrak{M}(v_{\bar{r}}^{(n-1)}, X_n)} f(X_n + t\mathbf{v}_{\bar{r}}^{(n-1)}) .$$

Если же точке X_n не соответствует тот же базис, что точке X_{n-1}, то для X_n повторяем весь процесс с первым вектором соответствующего базиса аналогично тому, как это было у точки X_1. Тогда справедлива

Теорема 2. *Последовательность $\{X_n\}_{n=1}^{\infty}$, построенная по описанному выше способу, является сходящейся; если обозначить $X_0 = \lim_{n\to\infty} X_n$, то $f(X_0) = \min_{X \in \mathfrak{M}} f(X)$.*

Параграф 4 посвящается применению изложенной теории к случаю, когда $f(X)$ является суммой линейной формы и положительно-определенной квадратичной формы.

В параграфе 5 показано применение изложенного метода на примере.

Summary

AN APPROXIMATIVE METHOD FOR NONLINEAR PROGRAMMING

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In the present paper an iterative process is formulated for finding the minimum of a strictly convex function $f(X)$ on the set $\mathfrak{M}$ of all nonnegative solutions of a system of linear equations

$$\sum_{j=1}^{m+k} a_{ij}x_j = b_i, \quad i = 1, \ldots, m, \; k \geqq 1 \tag{1}$$

of rank m. We suppose that $\mathfrak{M} \neq \emptyset$.

Before formulating our iterative process we must introduce some concepts.

By a vector of the set $\mathfrak{M}$ we mean a solution of the system of linear homogenous equations

$$\sum_{j=1}^{m+k} a_{ij} u_j = 0 \,, \quad i = 1, \dots, m \,. \tag{2}$$

We shall suppose that $\lim_{t \to \infty} f(X + t\mathbf{v}) = +\infty$ for every vector $\mathbf{v} = (v_1, \dots, v_{m+k})$ of $\mathfrak{M}$ such that $v_i \geqq 0$, $i = 1, \dots, m + k$, $\sum_{i=1}^{m+k} v_i > 0$ for every $X \in \mathfrak{M}$. Under this assumption the existence of the finite minimum of $f(X)$ on $\mathfrak{M}$ is proved (sec. 1).

A non-zero vector $\mathbf{u} = (u_1, \dots, u_{m+k})$ of $\mathfrak{M}$ is said to be *basic* if there are indices $i_1, \dots, i_{k-1}$ such that $u_{i_j} = 0$, $j = 1, \dots, k - 1$.

Let $X = (x_1, \dots, x_{m+k})$ be a point of $\mathfrak{M}$ for which $x_{i_j} = 0$, $j = 1, \dots, k - r$, $0 \leqq r \leqq k$. Then a *basis corresponding to* X is a set of k linearly independent basic vectors $\mathbf{v}_1, \dots, \mathbf{v}_k$ ($v_i = (v_{i1}, \dots, v_{i,m+k})$, $i = 1, \dots, k$) such that

1. there exist r vectors $\mathbf{v}_{\nu_1}, \dots, \mathbf{v}_{\nu_r}$ in the basis for which $v_{\nu_s, i_j} = 0$, $s = 1, \dots, r$, $j = 1, \dots, k - r$.

2. if $\mathbf{v}_{\mu_1}, \dots, \mathbf{v}_{\mu_{k-r}}$ are all others vectors of this basis then to every i_j, $j = 1, \dots, k - r$ there exists exactly one μ_j such that $v_{\mu_j, i_j} > 0$, $j = 1, \dots, k - r$ and that $v_{\mu_j, i_s} = 0$ for $j \neq s$.

A point $X \in \mathfrak{M}$ is *minimal with respect to a vector* $\mathbf{v}$ of $\mathfrak{M}$ if for every real t such that $X + t\mathbf{v} \in \mathfrak{M}$ it is $f(X + t\mathbf{v}) \geqq f(X)$.

For the proof of the convergence we need

Theorem 3. *Let $f(X)$ be a convex function defined on $\mathfrak{M}$ and posessing continucus partial derivatives of the 1st and 2nd order with respect to all variables. Than a necessary and sufficient condition for the relation $f(X_0) = \min_{x \in \mathfrak{M}} f(X)$ is that the point X_0 be minimal with respect to all vectors of some corresponding basis.*

We are now in a position to formulate our iterative process. Let X_1 be an arbitrary point of $\mathfrak{M}$, $\mathbf{u}_1, \dots, \mathbf{u}_k$ the vectors of a corresponding basis. Let us put $X_2 = X_1 + t_1 \mathbf{u}_1$ where t_1 is uniquely determined by the condition

$$f(X_1 + t_1 \mathbf{u}_1) = \min_{t \in \mathfrak{M}(u_1, X_1)} f(X + t\mathbf{u}_1)$$

($\mathfrak{M}(\mathbf{u}_1, X_1)$ is the set of all real t for which $X_1 + t\mathbf{u}_1 \in \mathfrak{M}$). If the same basis corresponds also to X_2 we put $X_3 = X_2 + t_2 \mathbf{u}_2$ determining t_2 by the condition

$$f(X_2 + t_2 \mathbf{u}_2) = \min_{t \in \mathfrak{M}(u_2, X_2)} f(X_2 + t\mathbf{u}_2) \,.$$

If this is not the case we proceed for X_2 with the first vector of a corresponding basis as for X_1.

Let us now suppose we have already constructed the X_n. Let $\mathbf{v}_1^{(n-1)}, \ldots$ $\ldots, \mathbf{v}_k^{(n-1)}$ be the basis corresponding to X_{n-1} and let $X_n = X_{n-1} + t_{n-1}\mathbf{v}_r^{(n-1)}$ $(1 \leqq r \leqq k)$; if this basis corresponds also to X_n we put $X_{n+1} = X_n + t_n\mathbf{v}_{\bar{r}}^{(n-1)}$ where $\bar{r} \equiv r + 1 \pmod{k}$, $1 \leqq \bar{r} \leqq k$ and determine t_n by the condition

$$f(X_n + t_n\mathbf{v}_{\bar{r}}^{(n-1)} (= \min_{t \in \mathfrak{M}(v_{\bar{r}}^{(n-1)}, X_{n-1})} f(X_n + t\mathbf{v}_{\bar{r}}^{(n-1)}) .$$

In the contrary case we find a basis corresponding to X_n and construct X_{n+1} from X_n as X_2 from X_1.

Theorem 4. *The sequence* $\{X_n\}_{n=1}^{\infty}$ *constructed by the above described method is convergent. Denoting* $X_0 = \lim_{n\to\infty} X_n$ *we have* $f(X_0) = \min_{x \in \mathfrak{M}} f(X)$.

In sec. 4 the case is studied where $f(X)$ is a sum of a linear form and a positive definite quadratic form.

In sec. 5 a numerical exemple is given.