пенями свободы; наоборот, значение t будет (независимо от постоянных λ_j) значимым, если оно значимо относительно распределения Стьюдента с числом степеней свободы $\min\limits_{1 \leq j \leq k} m_j$. Такое положение наступает, напр. тогда, если x является линейной комбинацией k статистик, вычисленных из k независимых выборок, дисперсии которых отличаются друг от друга неизвестным образом. Самым известным случаем является разность двух независимых выборочных средних $\bar{x} - \bar{y}$.

Из теоремы следует между прочим и то, что доверите ьный интервал $x \pm ts$, где s^2 имеет структуру (2) и t взято из распределения Стьюдента с числом степеней свободы $\min\limits_{1 \leq j \leq k} m_j$, покроет среднее значение μ с вероятностью, большей чем та, которой соответствует использованное значение t.

Summary

INEQUALITIES FOR THE GENERALISED STUDENT'S DISTRIBUTION AND THEIR APPLICATIONS

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In this paper the following theorem is proved:

Theorem. *Let x be a normally (μ, σ^2) distributed random variable, and let s^2 be an estimate of σ^2 with the structure*

$$s^2 = \sigma^2 \sum_{j=1}^{k} \frac{\lambda_j}{m_j} \chi_j^2(m_j), \quad \lambda_j \geqq 0, \quad \sum_{j=1}^{k} \lambda_j = 1, \tag{2}$$

where the λ_j's are unknown constants and the random variables $\chi_j^2(m_j)$ have chi-square distributions with m_j degrees of freedom and are independent of one another and of x; let us choose arbitrary bounds $t' \leqq 0 \leqq t''$.

Under these conditions the probability P of the event

$$t' \leqq \frac{x - \mu}{s} < t'', \qquad t' \leqq 0 \leqq t'' \tag{14}$$

lies between the limits $P_\nu \leqq P \leqq P_m$, where P_m, P_ν denote the probability of the event (14) under the condition that $(x - \mu)/s$ has Student's distribution with m, ν degrees of freedom, respectively, $m = m_1 + m_2 + \ldots + m_k$ and ν may be any arbitrary integer $\nu \leqq \min\limits_{1 \leq j \leq k} \frac{m_j}{\lambda_j}$, for example, $\nu = \min\limits_{1 \leq j \leq k} m_j$.

This result may be applied to the testing of the null hypothesis that a statistic x has a prescribed mean value μ_0 provided that the estimate s^2 of the

variance of x has the structure (2). Indeed, the observed value $t = \frac{x - \mu_0}{s}$ is (independently of the λ_j's) not significant, if it is not significant with respect to the Student's distribution with $m = m_1 + m_2 + \ldots + m_k$ degrees of freedom; conversely, t is (independently of the λ_j's) significant if it is significant with respect to the Student's distribution with $\min_{1 \leq j \leq k} m_j$ degrees of freedom. Such a situation arises, for example, if x is a linear combination of k statistics evaluated from k independent samples, whose variances differ in an unknown manner. The most famous case is the difference of two independent sample means $\bar{x} - \bar{y}$.

From the theorem it also follows, that the confidence interval $x \pm ts$, where s^2 has the structure (2) and t is taken from the Student's distribution with $\min_{1 \leq j \leq k} m_j$ degrees of freedom, will contain the mean value μ with a probability greater than that which corresponds to the used t.