вок. Тогда Λ будет линейным пространством, образованным всеми T_λ для всех систем транзитивности S_λ.

В последующих теоремах изучается множество несобственных избранных точек, собственных избранных прямых и избранных гиперплоскостей. Для полноты и в целях использования в третьей части приводятся некоторые избранные точки и избранные множества, являющиеся обобщением некоторых избранных (замечательных) точек треугольника, и обсуждаются их свойства. В заключение исследуется изогональное сродство в симплексе и доказывается, между прочим, что изогонально сопряженные точки являются фокусами гиперквадрик вращения, касающихся всех $(n - 1)$-мерных граней симплекса.

Summary

GEOMETRY OF THE SIMPLEX IN E_n (2nd part)

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In the present paper (first part is published in Časopis pro pěstování matematiky *79* (1954), 297—320) we begin with the definition of an admissible mapping in the Euclidean n-space E_n. We say that φ is an admissible mapping of degree m ($m \geqq 0$ an integer) in E_n if for every ordered set of $m + 1$ points $A_1, \ldots, A_{m+1} \in E_n$ $\varphi(A_1, \ldots, A_{m+1})$ is a subset of $\bar{E}_n$ (i. e. E_n completed with improper points) such that, for every isometric mapping T of $\bar{E}_n$,

$$\varphi(TA_1, \ldots, TA_{m+1}) = T\,\varphi(A_1, \ldots, A_{m+1}) \, .$$

Let Σ be a set of fixed points $A_1, \ldots, A_{m+1} \in E_n$. We say that a set $M \subset \bar{E}_n$ is a distinguished set of Σ if there exists an admissible mapping φ of degree m such that

$$M = \varphi(A_{i_1}, \ldots, A_{i_{m+1}})$$

for every permutation $i_1, \ldots, i_{m+1}$ of $1, \ldots, m + 1$. A set is a distinguished set of a simplex in E_n if it is a distinguished set of its $n + 1$ vertices.

Theorems 16—20 show that the set of all proper distinguished points (i. e. distinguished one-point-sets) of the simplex is a linear space Λ formed the following way:

The group of all automorphisms of the simplex (i. e. the group of all isometric transforms in E_n preserving the set of all vertices $O_1, \ldots, O_{n+1}$ of the simplex) is isomorphic to a permutation group of indices $1, \ldots, n + 1$, since for every such transform T there exists exactly one permutation $i_1, \ldots, i_{n+1}$ such that $TO_k = O_{i_k}$. Let us call transitivity systems of the vertices the sets S_λ of those vertices of the simplex whose indices belong to one of the transitivity systems of the permutation group. Then Λ is the linear space generated by all the centres of gravity T_λ of the transitivity systems S_λ (theorem 20).

In the following theorems the sets of all improper distinguished points, proper distinguished lines, and distinguished hyperplanes are studied. For the sake of completeness and for the use in the third part some special distinguished points and distinguished sets are mentioned.

Finally the isogonal relationship in the simplex is studied. In theorem 28 is proved that isogonally associated points are foci of hyperquadrics of rotation touching all the faces of the simplex.