II. Nenastane případ I. Bez újmy obecnosti můžeme předpokládat, že $x_m \in u\,D(y_m)$, $y_m \neq a$, body y_m jsou navzájem různé a takové, že $y_m \in I_{n_m}$ s nejmenším možným indexem n_m. Pak jest $x_m \neq y_m$ pro všecka m. Vskutku, kdyby $x_m = y_m$ pro určitý index m, pak by $n_m = 0$, takže $y_m = a$; tato úvaha vyplývá odtud, že ke každému bodu $y \in I_n$, $n > 0$, existuje bod $y' \in I_{n-1}$ takový, že $y \in D(y') \subset u\,D(y')$ a z předpokladu, že n_m je nejmenší index s vlastností $x_m \in u\,D(y_m)$, $y_m \in I_{n_m}$. Uvažme nyní, že $y_m \in J$, takže

$$x_m \in u\,D(y_m) = y_m \cup \bigcup_{n \geq 2k} E\,(\alpha, \beta, n, \varphi(y_m))$$

a jelikož $x_m \neq y_m$, jest $x_m = (\alpha_m, \beta_m, n_{(m)}, \varphi(y_m))$. Poněvadž $y_m \neq y_n$, jest také $\varphi(y_m) \neq \varphi(y_n)$ pro $m \neq n$. Podle poznámky na str. 62 není $\lim x_m = x$, čímž dostáváme spor. Případ II nemůže nastati.

Dokázali jsme, že $W'(a) = u\,W'(a)$. Zřejmě také $W'(a) = v\,W'(a)$, neboť U-modifikace zachovává u-uzavřené množiny. Poněvadž $D(y) \subset u\,D(y)$, jest $W(a) \subset W'(a)$, tudíž $v\,W(a) \subset v\,W'(a)$. Na druhé straně jest $D(y) \subset W(a)$, tedy $u\,D(y) \subset u\,W(a)$, z čehož vyplývá, že $W'(a) \subset u\,W(a) \subset v\,W(a) \subset v\,W'(a) = W'(a)$, to je $W'(a) = v\,W(a)$. Vztah $W'(a) \subset V(a)$ vyplývá z (*).

*

Regular space, on which evevy continuous functions is constant.

(Summary of the preceding article.)

The present paper concerns the solution of Urysohn's problem of the existence of regular topological spaces R in which every continuous real-valued function is constant. There exists such a space R of arbitrary uncountable cardinal number. First of all it is proved that every real-valued continuous function defined on the space $P = A \times B - (\alpha^*, \beta^*)$ is constant, where A and B are two compact L-spaces of infinite cardinal numbers $\mathfrak{a} < \mathfrak{b}$, in which every point except two, α^* and β^*, is isolated. Now, let Q be the set of all quadruples $(\alpha, \beta, n, \gamma)$, $\alpha \in A$, $\beta \in B$, n positive integer, $\gamma \in B$, $(\alpha, \beta) \neq (\alpha^*, \beta^*)$, where two identifications hold:

I. Tychonoff's identification

$(\alpha, \beta^*, n, \gamma) = (\alpha, \beta^*, n + 1, \gamma)$ for all odd integers $n = 1, 3, \ldots$

$(\alpha^*, \beta, n, \gamma) = (\alpha^*, \beta, n + 1, \gamma)$ for all even integers $n = 2, 4, \ldots$

II. $(\alpha^*, \beta, 1, \gamma) = (\alpha^*, \beta, 1, \gamma')$ for all $\gamma \in B$, $\gamma' \in B$.

Let the point-set $\underset{(\alpha,\beta)}{E}\,(\alpha, \beta, n_0, \gamma_0) - (\alpha^*, \beta^*, n_0, \gamma_0)$ be homeomorphic

with the space P for all integers $n_0 > 0$ and all $\gamma_0 \in B$; let $\gamma = \varphi(z)$ be a one-to-one correspondence between J and B, where J denotes all isolated points $z = (\alpha, \beta, n, \gamma)$, $\alpha \neq \alpha^*$ and simultaneously $\beta \neq \beta^*$; let us define $\lim z_n = z$, where $z_n = (\alpha_n, \beta_n, n, \varphi(z))$. Then, Q is a regular **L**-space, on which every real continuous function is constant, but in which the neighbourhoods need not be open. The construction of the space R makes use of Čech's idea of U-modification of topology in Q, in which the neighbourhoods are open.

Urysohn's problem was solved by the present author in September 1946. Towards the end of the same year a copy of the Annals of Mathematics 47 (1946), arrived in Prague, in which Edwin Hewitt presents the following solution:

Let $\aleph$ be an infinite cardinal number which is not the sum of $\aleph_0$ cardinal numbers smaller than $\aleph$. Then there exists a regular space R of cardinal number $\aleph$ in which every continuous real-valued function is constant.

Both solutions, Hewitt's and mine, are independent on one another. Hewitt's solution rightly claims priority, for it was sent to press in October 1945. In spite of this, I have thought it would be of some use to submit my own solution, as it is more general presenting the construction of the space R for an arbitrary uncountable cardinal number. Likewise my method of construction and proof differ from Urysohn's and Hewitt's.