

## Werk

Titel: 6 Local and global Hirzebruch-Riemann-Roch Formula for coherent sheaves on Vi-man...

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52 R. Blache

which is generated by  $\hat{\mathscr{S}}$  and  $\hat{\mathscr{S}}'$ . I claim that  $\pi_*^G \hat{\mathscr{S}}'' = \mathscr{S}$ ; the proof is left to the reader. Now let  $\hat{\mathscr{S}}^{(t)}$  be the maximal G-invariant subsheaf of  $(\hat{\mathscr{S}})^{\vee\vee}$  such that  $\pi_*^G (\hat{\mathscr{S}}^{(t)}) = \mathscr{S}$ . By the above arguments,  $\hat{\mathscr{S}}^{(t)}$  exisits; by [Fi 76], p. 36,  $\hat{\mathscr{S}}^{(t)}$  is coherent. I claim that  $\hat{\mathscr{S}}^{(t)}$  is the terminal object of  $Coh_S^G(Y, y)$ ; the proof is left to the reader.  $\square$ 

**(5.17) Lemma.** Let X be a projective Vi-manifold. Let  $\mathscr{F}$  be a coherent sheaf on X. Assume that  $\mathscr{F}|_{(X,x)}$  is V-free for all  $x \in \operatorname{Sing} X$ . Hence there is a Zariski-open subset  $U \subseteq X$ , such that  $\mathscr{F}|_U$  is locally V-free. Let  $\sigma: \tilde{X} \to X$  be a good resolution of X with  $\sigma(E) = \operatorname{Sing} X$ , such that there is a coherent sheaf  $\widetilde{\mathscr{F}}$  on  $\widetilde{X}$  with  $(\sigma_*\widetilde{\mathscr{F}})^{\vee\vee} = \mathscr{F}$  and with  $\widetilde{\mathscr{F}}|_{\widetilde{U}}$  locally free. For every  $x \in \operatorname{Sing} X$ , set  $\widehat{\mathscr{F}}_x := (\pi_x^*\mathscr{F})^{\vee\vee}$ . Then  $c(\mathscr{F}, \{\widehat{\mathscr{F}}_x\}) + \sum_{x \in \operatorname{Sing} X} c(x, (\mathscr{F}, \widetilde{\mathscr{F}})) = c(\widetilde{\mathscr{F}})$ , where  $c(x, (\mathscr{F}, \widetilde{\mathscr{F}}))$  is as in (3.10) and  $c(\mathscr{F}, \{\widehat{\mathscr{F}}_x\})$  as in (5.9).

This Lemma will be used in Sect. 6.

Proof. By Theorem 5.9,  $c(\mathcal{F}, \{\hat{\mathcal{F}}_x\}) + \sum_{x \in \operatorname{Sing} X} c(x, \mathcal{F}, \hat{\mathcal{F}}_x, \sigma^* \mathcal{F}) = c(\sigma^* \mathcal{F})$ . By arguments, similar to those used in (5.5), (5.6), and (5.7), using homomorphisms  $\tilde{\mathcal{F}} \to (\sigma^* \mathcal{F})^{\vee\vee} \otimes \mathcal{O}_{\tilde{X}}(lE)$  and  $\sigma^* \mathcal{F} \to (\sigma^* \mathcal{F})^{\vee\vee} \to (\sigma^* \mathcal{F})^{\vee\vee} \otimes \mathcal{O}_{\tilde{X}}(lE)$  (for some  $l \gg 0$ ) which are isomorphisms outside E, it can be seen that there are natural classes  $c((\tilde{X}, E_x), (\tilde{\mathcal{F}}, \sigma^* \mathcal{F})) \in H^*_{dRc}((\tilde{X}, E_x), \mathbb{C})$  (where  $x \in \operatorname{Sing} X$ ), such that  $c(\tilde{\mathcal{F}}) + \sum_{x \in \operatorname{Sing} X} c((\tilde{X}, E), (\tilde{\mathcal{F}}, \sigma^* \mathcal{F})) = c(\sigma^* \mathcal{F})$  in  $H^*_{dR}(\tilde{X}, \mathbb{C})$ . Then it is a consequence of the definition of the local Chern classes  $c(x, (\mathcal{F}, \tilde{\mathcal{F}}))$  (see (3.10)) that  $c(x, (\mathcal{F}, \tilde{\mathcal{F}})) = c(x, \mathcal{F}, \hat{\mathcal{F}}_x, \sigma^* \mathcal{F}) - c((\tilde{X}, E), (\tilde{\mathcal{F}}, \sigma^* \mathcal{F}))$ ; from this our claim follows.  $\square$ 

# 6 Local and global Hirzebruch-Riemann-Roch Formula for coherent sheaves on Vi-manifolds; open problems

Using the Chern classes introduced in Sect. 5, we prove the HRR Theorem for coherent sheaves on projective Vi-manifolds, cf. Theorem 6.1. The method is to prove a local HRR Theorem first, cf. Theorem 6.2; this, in turn, is proved using a globalization argument which reduces the HRR Theorem to the situation that the Vi-manifold under consideration is *globally* the quotient by a finite group. In sections (6.6) to (6.8) we briefly discuss some open problems.

**(6.1) Theorem.** Let X be a projective Vi-manifold, let  $\mathcal{S}$  be a coherent sheaf on X. Fix liftings  $\hat{\mathcal{S}}_x \in \operatorname{Coh}_S^{G_x}(Y,y)$  for all  $x \in \operatorname{Sing} X$ , where  $(Y,y) \to (X,x)$  is the local smoothing covering of X at X. Then

$$\chi(X,\mathcal{S}) = \chi_{\text{orb}}(X,(\mathcal{S},\{\hat{\mathcal{S}}_x\})) + \sum_{x \in \text{Sing } X} \mu_{X,x}((\mathcal{S},\hat{\mathcal{S}}_x)).$$

Remark.

i) By Proposition 5.13,  $K_{VV}((X,x)) \cong K_{VC}((X,x))$ , and hence  $\mu_{X,x}: K_{VV}((X,x)) \to \mathbb{Q}$  induces a group homomorphism  $\mu_{X,x}: K_{VC}((X,x)) \to \mathbb{Q}$ ; thus  $\mu_{X,x}((\mathcal{S},\hat{\mathcal{S}}_x)) = \mu_{X,x}([\hat{\mathcal{S}}_x])$  is well defined.

ii) The meaning of  $\chi_{\text{orb}}(X, (\mathcal{S}, \{\hat{\mathcal{S}}_x\}))$  should be clear; it is defined as in (3.4), applied to the Chern classes  $c_1(\mathcal{S}, \{\hat{\mathcal{S}}_x\}), \ldots, c_n(\mathcal{S}, \{\hat{\mathcal{S}}_x\})$ , which are defined in (5.9).

The proof of the Theorem is given in (6.5).

**(6.2) Theorem.** Let  $(\tilde{X}, E) \stackrel{\sigma}{\to} (X, x)$  be a good resolution of an isolated quotient singularity. Let  $(Y, y) \stackrel{\pi}{\to} (X, x)$  be the local smoothing covering. Let  $\mathcal{S}$  be a coherent sheaf on (X, x). Assume that the pair  $((X, x), \mathcal{S})$  can be realized on a projective model. Let  $\hat{\mathcal{S}}$  be a fixed lifting of  $\mathcal{S}$  to (Y, y), set  $\tilde{\mathcal{S}} := \sigma^* \mathcal{S}$ . Set

$$\chi(x,(\mathcal{S},\tilde{\mathcal{S}})):=h^0(\mathcal{B})-h^0(\mathcal{A})+\sum_{1\leq i\leq n-1}(-1)^i\cdot h^0(R^i\sigma_*\tilde{\mathcal{S}})\,,$$

where  $0 \to \mathscr{A} \to \mathscr{S} \to \sigma_* \tilde{\mathscr{S}} \to \mathscr{B} \to 0$  is the natural exact sequence with  $\sup \mathscr{A} \cup \sup \mathscr{B} \subseteq \{x\}$ . Then

$$\chi(x,(\mathscr{S},\tilde{\mathscr{S}})) = -\chi_{\mathrm{orb}}(x,(\mathscr{S},\hat{\mathscr{S}},\tilde{\mathscr{S}})) + \mu_{X,x}((\mathscr{S},\hat{\mathscr{S}})).$$

Remark.

- i) This Theorem should be seen as a *local HRR Theorem*, similar to Theorem 3.16. There is also another generalization of Theorem 3.16 which we mention without proof: Let  $(\tilde{X},E) \stackrel{\sigma}{\to} (X,x)$  be a good resolution of an isolated quotient singularity, let  $\tilde{\mathscr{F}}$  be a reflexive sheaf on  $\tilde{X}$  and set  $\mathscr{F}:=(\sigma_*\tilde{\mathscr{F}})^{\vee\vee}$ . For every  $x\in \operatorname{Sing} X$  we fix the canonical lifting  $\hat{\mathscr{F}}_x:=(\pi_x^*\mathscr{F})^{\vee\vee}$ , where  $\pi_x$  is the local smoothig covering of X at x. Then it is possible to define local Chern classes  $c_i(x,(\mathscr{F},\tilde{\mathscr{F}}))=c_i(x,(\mathscr{F},\hat{\mathscr{F}}_x,\tilde{\mathscr{F}}))\in H^{2i}_{dR}((\tilde{X},E),\mathbb{C})$  in a natural way. Moreover, under the assumption that the pair  $((X,x),\mathscr{F})$  can be realized by a projective model, the local HRR Formula  $\chi(x,(\mathscr{F},\tilde{\mathscr{F}}))=-\chi_{\operatorname{orb}}(x,(\mathscr{F},\tilde{\mathscr{F}}))+\mu_{X,x}((\mathscr{F},\hat{\mathscr{F}}_x))$  holds.
- ii) The rational number  $\chi_{\text{orb}}(x,(\mathcal{S},\hat{\mathcal{S}},\tilde{\mathcal{S}}))$  is defined as in (3.13), with respect to the local Chern numbers  $c_i(x,(\mathcal{S},\hat{\mathcal{S}},\tilde{\mathcal{S}})) \in H^{2i}_{dRc}((\tilde{X},E),\mathbb{C})$ , which are defined in (5.8).
- iii) Since the statement of the Theorem is of a purely local nature, it is only natural to conjecture that the assumption that the pair  $((X,x),\mathcal{S})$  can be realized projectively is superfluous and artificial.

The proof of the Theorem is given in (6.4).

**(6.3) Lemma.** Let X be a projective Vi-manifold, let  $\mathcal{F}$  be a coherent sheaf on X, assume that  $\mathcal{F}|_{(X,x)}$  is V-free for all  $x \in \operatorname{Sing} X$ . Then  $\chi(X,\mathcal{F}) = \chi_{\operatorname{orb}}(X,\mathcal{F}) + \sum_{x \in \operatorname{Sing} X} \mu_{X,x}(\mathcal{F})$ .

*Proof.* Let  $U\subseteq X$  be a Zariski-open subset such that  $\mathrm{Sing}\,X\subseteq U$  and such that  $\mathscr{F}|_U$  is locally V-free. Let  $\tilde{X}\stackrel{\sigma}{\to} X$  be a resolution of X and let  $\tilde{\mathscr{F}}$  be a coherent sheaf on  $\tilde{X}$  such that  $\tilde{\mathscr{F}}|_{\tilde{U}}$  is locally free and such that  $(\sigma_*\tilde{\mathscr{F}})^{\vee\vee}\cong\mathscr{F}$  and such that the induced map of germs  $(\tilde{X},\tilde{A})\to (X,A)$  is biholomorphic, where  $X=U\uplus A,\,\tilde{X}=\tilde{U}\uplus \tilde{A}$ . Hence, using Lemma 3.9, classical HRR,

54 R. Blache

Theorem 3.16, Lemma 5.17, and Proposition 3.14,

$$\chi(X, \mathcal{F}) = \chi(\tilde{X}, \tilde{\mathcal{F}}) + \sum_{x \in \text{Sing } X} \chi(x, \tilde{\mathcal{F}})$$

$$= \chi_{\text{orb}}(\tilde{X}, \tilde{\mathcal{F}}) + \sum_{x \in \text{Sing } X} (-\chi_{\text{orb}}(x, \tilde{\mathcal{F}}) + \mu_{X,x}(\mathcal{F}))$$

$$= \chi_{\text{orb}}(X, \mathcal{F}) + \sum_{x \in \text{Sing } X} \mu_{X,x}(\mathcal{F}). \quad \Box$$

### (6.4) Proof of Theorem 6.2.

- a) Since the pair  $((X,x),\mathcal{S})$  can be realized projectively, we can assume that X is a projective Vi-manifold such that  $\#\operatorname{Sing} X=1$  and such that  $\mathscr{S}$  is a coherent sheaf on X. Set  $G:=G_X$ .
- b) By [Ar 66], Theorem II.5.2, we can find an affine open neighbourhood U of x in X, an affine Vi-manifold V, a finite surjective locally biholomorphic map  $V \to U$ , an affine manifold W, an injection  $G \hookrightarrow \operatorname{Aut} V$ , such that  $V \cong W/G$ .
- c) Let  $V \hookrightarrow \bar{V}$  be a projective closure such that Sing  $V = \operatorname{Sing} \bar{V}$ , and let  $\bar{W}$  be the normalization of  $\bar{V}$  in the function field of W. Hence G acts on  $\bar{W}$ ,  $\bar{W}/G = \bar{V}$ , and

$$\begin{array}{ccc} W & \hookrightarrow & \bar{W} \\ \downarrow & & \downarrow \\ V & \hookrightarrow & \bar{V} \end{array}$$

is a commutative diagram. Clearly, the sheaf  $\mathscr S$  on X can be realized as a coherent sheaf on  $\bar V$  (restrict to U, lift to V, extend to  $\bar V$ ); let us denote this sheaf by  $\mathscr S$ , again.

d) Let  $\hat{\mathscr{S}}$  be a coherent G-sheaf on  $\bar{W}$  such that  $\pi^G_*\hat{\mathscr{S}}=\mathscr{S}$  (where  $\pi$  is the projection  $\bar{W}\to \bar{V}$ ) and such that  $\hat{\mathscr{S}}|_{(\bar{W},y)}$  is isomorphic to the given lifting of  $\mathscr{S}|_{(X,x)}$  to the smoothing covering of (X,x), for every  $y\in W\cap \operatorname{Fix} G$ . Choose a G-equivariant resolution  $0\to \hat{\mathscr{F}}_n\to\cdots\to\hat{\mathscr{F}}_0\to \hat{\mathscr{S}}\to 0$  of  $\hat{\mathscr{S}}$  over  $\bar{W}$ , as in Lemma 5.2;  $\hat{\mathscr{F}}_i|_W$  is locally free for every  $i=0,\ldots,n$ . We push down by  $\pi^G_*$  to an exact sequence  $0\to \mathscr{F}_n\to\cdots\to \mathscr{F}_0\to \mathscr{S}\to 0$  over  $\bar{V}$ ,  $\mathscr{F}_i|_V$  is locally V-free for every  $i=0,\ldots,n$ .

e) Claim. 
$$\chi(\bar{V}, \mathcal{S}) = \chi_{\text{orb}}(\bar{V}, (\mathcal{S}, \{\hat{\mathcal{S}}_x \mid x \in \text{Sing } V\})) + \sum_{x \in \text{Sing } V} \mu_{X,x}((\mathcal{S}, \hat{\mathcal{S}}_x))$$

*Proof.* Using Proposition 2.11 and Theorem 3.5, it is sufficient to show that  $\chi(\bar{V}, \mathscr{F}_i) = \chi_{\text{orb}}(\bar{V}, \mathscr{F}_i) + \sum_{x \in \text{Sing } V} \mu_{X,x}(\mathscr{F}_i)$  for all i = 0, ..., n. But this is Lemma 6.3.

f) Let  $\sigma: \tilde{V} \to \bar{V}$  be the resolution which is induced by the given resolution  $\sigma: (\tilde{X}, E) \to (X, x)$ . Then

$$\chi(x,(\mathcal{S},\tilde{\mathcal{S}})) = \frac{1}{N} \cdot (\chi(\bar{V},\mathcal{S}) - \chi(\tilde{V},\tilde{\mathcal{S}}))$$

$$= \frac{1}{N} \cdot (\chi_{\text{orb}}(\bar{V},(\mathcal{S},\{\hat{\mathcal{S}}_x\})) + \sum_{x \in \text{Sing } V} \mu_{X,x}((\mathcal{S},\hat{\mathcal{S}})) - \chi_{\text{orb}}(\tilde{V},\tilde{\mathcal{S}}))$$

$$= \frac{1}{N} \cdot \left( -\sum_{x \in \text{Sing } V} \chi_{\text{orb}}(x, (\mathcal{S}, \hat{\mathcal{S}}, \tilde{\mathcal{S}})) + \sum_{x \in \text{Sing } V} \mu_{X,x}((\mathcal{S}, \hat{\mathcal{S}})) \right)$$

$$= -\chi_{\text{orb}}(x, (\mathcal{S}, \hat{\mathcal{S}}, \tilde{\mathcal{S}})) + \mu_{X,x}((\mathcal{S}, \hat{\mathcal{S}})),$$

where we used Lemma 3.8, Claim e, classical HRR, and Proposition 3.14.

- **(6.5)** Proof of Theorem 6.1. This is now a direct consequence of Theorem 6.2, classical HRR, Lemma 3.8, Theorem 5.9, and Proposition 1.18.  $\Box$
- (6.6) In the following sections we want to discuss some problems which remain open. Obviously, most of the results presented in this article are not of a final nature. They want to be generalized in at least two directions:
- (a) So far as definitions of Chern classes or HRR type Theorems are concerned, all projectivity assumptions and all "isolated singularities" assumptions should be eliminated. Everything should work out for coherent sheaves on arbitrary compact complex V-manifolds.
- (b) One should try to carry over everything to normal complex-projective varietes with isolated singularities, or maybe to an even more restricted class of singularities (but not as restricted as isolated quotient singularities).
- (6.7) We want to give a few comments on item (6.6.a). There seem to be two promising ways to attack this problem. The first one is to start with a generalization of Theorem 3.17 to nonisolated singularities: since Theorem 3.17 expresses our correction terms  $\mu_{X,x}$  by an Atiyah–Bott type formula, derived from a fixed point formula for automorphisms with isolated fixed points, and since corresponding formulas exist for automorphisms with nonisolated fixed points, one may derive from these formulas how  $\mu_{X,x}$  has to look in case of nonisolated singularities; most likely  $\mu_{X,x}$  has to be substituted by differential forms of respective top degree on the strata of Sing X.

The second way to attack (6.6.a) may be to use ideas of H.I. Green, N.R. O'Brian, D. Toledo, and Y.L.L. Tong, as presented in [T-T 86]: There, an approach to the definition of Chern classes for coherent sheaves on compact complex manifolds is given which is "soft" enough to give a definition, and which is "rigid" enough to enable one to prove HRR for coherent sheaves on compact complex manifolds. Moreover, it seems to be "local"enough to survive the additional local fundamental groups  $G_x$  which come together with (compact) complex V-manifolds. Notice that the main difficulty which is overcome by [T-T 86] is similar to the difficulty we face in our situation: the former is the lack of a global locally free resolution of a given coherent sheaf on a compact complex manifold (this question seems to be open), the latter is the lack of a global locally V-free V-exact resolution of a coherent sheaf on a compact complex V-manifold (even in case the V-manifold is projective; this question is also open).

In this paper, we often use both the resolution of singularities and the local smoothing covering to construct objects and to prove results. However, as far as item (6.6.a) is concerned, it may be possible to eliminate the need

56 R. Blache

of resolution of singularities (unless resolution of singularities is of interest for its own sake, as e.g. in Sect. 4).

(6.8) Problem (6.6.b) seems to be even more difficult than problem (6.6.a). Notice that even the following question (which is a very special case of (6.6.b)) is open: Let X be a normal projective surface (since then codim Sing  $X \ge 2$ , Sing X only consists of finitely many points). Let  $\mathscr{F}$  be a reflexive sheaf of rank two on X. What is  $c_2(\mathscr{F})$ ? When  $c_2(\mathscr{F})$  is defined: what is the HRR formula then? (In case rank  $\mathscr{F} = 1$ , HRR is worked out in [B1 93a].) Certainly,  $c_2(\mathscr{F})$  has to be a real number (even a rational one?).

If the Wahl-Conjecture (see (4.8)) is true, then there is a candidate for  $c_2(\mathscr{F})$ : Let  $\tilde{X} \stackrel{\sigma}{\to} X$  be a resolution such that there is a locally free sheaf  $\tilde{\mathscr{F}}$  on  $\tilde{X}$  such that  $(\sigma_*\tilde{\mathscr{F}})^{\vee\vee} = \mathscr{F}$  and such that  $\sigma(E) = \operatorname{Sing} X$  (this exists, see (3.7)). Then

$$c_2(\mathscr{F}) := c_2(\tilde{\mathscr{F}}) - \sum_{x \in \text{Sing } X} \tilde{c}_2(x, \tilde{\mathscr{F}})$$

is a natural guess, see (4.8) for the definition of Wahl's  $\tilde{c}_2(x, \tilde{\mathscr{F}})$ . Interpretating definition (4.8) as a *splitting principle*, one should then try to globalize this to a definition of Chern classes of reflexive sheaves on normal projective varieties; the local smoothing coverings of V-manifolds could then be replaced by the collection of all generically finite mappings  $Y \to X$ , where Y is any complex-projective manifold.

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