

## Werk

**Titel:** 0 Introduction and statement of results.

**Jahr:** 1993

**PURL:** https://resolver.sub.uni-goettingen.de/purl?266833020\_0212|log72

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# On a borderline class of non-positively curved compact Kähler manifolds

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Received November 26, 1991; in final form June 16, 1992

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### 0 Introduction and statement of results

Let M be a compact complex manifold. Denote by  $\mathscr{F}(M)$  the space of all Kähler metrics on M with non-positive holomorphic bisectional curvature. Since the summation of two such metrics still has the same curvature property,  $\mathscr{F}(M)$  forms a convex subset in  $\mathscr{C}(M)$ , the linear span of the space of all Kähler metrics on M.

**Definition.** M is said to be semi-rigidly non-positively curved, or simply semi-rigid, if  $\mathcal{F}(M)$  is not empty, and its linear span in  $\mathcal{C}(M)$  is finite dimensional.

It is not hard to see that for a finite unbranched cover  $\pi: M \to N$ , M is semi-rigid if and only if N is so; and for a product manifold  $M = M_1 \times M_2$ , M is semi-rigid if and only if both  $M_1$  and  $M_2$  are so.

Apparently, if there is a metric g on M which has strictly negative holomorphic bisectional curvature at a point  $x \in M$ , then any small perturbation of g near x is also in  $\mathcal{F}(M)$ , so M cannot be semi-rigid. In other words, semi-rigidity is likely to occur only when the cotangent bundle  $T_M^*$  is semi-ample but not ample in certain strong way, which could give lots of flat directions for the curvature of any g in  $\mathcal{F}(M)$ . This would tie the elements of  $\mathcal{F}(M)$  together.

<sup>\*</sup> Research supported by NSF Grant DMS-91-05185 and Duke University