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On a borderline class of non-positively curved compact Kähler manifolds

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0 Introduction and statement of results

Let M be a compact complex manifold. Denote by $\mathscr{F}(M)$ the space of all Kähler metrics on M with non-positive holomorphic bisectional curvature. Since the summation of two such metrics still has the same curvature property, $\mathscr{F}(M)$ forms a convex subset in $\mathscr{C}(M)$, the linear span of the space of all Kähler metrics on M.

Definition. M is said to be *semi-rigidly non-positively curved*, or simply *semi-rigid*, if $\mathscr{F}(M)$ is not empty, and its linear span in $\mathscr{C}(M)$ is finite dimensional.

It is not hard to see that for a finite unbranched cover $\pi\colon M \to N$, M is semi-rigid if and only if N is so; and for a product manifold $M = M_1 \times M_2$, M is semi-rigid if and only if both M_1 and M_2 are so.

Apparently, if there is a metric g on M which has strictly negative holomorphic bisectional curvature at a point $x \in M$, then any small perturbation of g near x is also in $\mathscr{F}(M)$, so M cannot be semi-rigid. In other words, semi-rigidity is likely to occur only when the cotangent bundle T_M^* is semi-ample but not ample in certain strong way, which could give lots of flat directions for the curvature of any g in $\mathscr{F}(M)$. This would tie the elements of $\mathscr{F}(M)$ together.

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