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Persson's list of singular fibers for a rational elliptic surface

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0. Introduction

This work is the product of my attempts to understand the list of possible singular fibres which can occur on a rational elliptic surface with section, which has recently been produced by U. Persson [P]. In his work, he constructs all the possible configurations, and proves the impossibility of the ones which cannot exist, by using very geometric arguments; these all boil down to various constructions involving plane curves of low degree, and distinguished points on these curves, having prescribed singularities. It is an impressive illustration of the beauty of the geometry of plane curves, and any interested reader will have a lot of fun studying the necessary constructions.

In this article I will concentrate on more combinatorial criteria for the existence of a rational elliptic surface with prescribed singular fibres. In this way one is able to reproduce Persson's list, and it is hoped that the two approaches complement and reinforce one another. In addition, one obtains a completely different construction for the surfaces which exist, and gives an independent verification for the final list.

One can take a rational elliptic surface with section S, and blow down all components of fibers which do not meet S; one obtains an elliptic surface with a finite number of rational double points. The classification of the rational double point configurations which can be obtained this way has been done: the reader should consult [D], [L], [T], and [U]. This classification ignores the difference between several fiber types: I_0, I_1, and II contribute no rational double point, I_2 and III both give an A_1 singularity, and I_3 and IV both

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