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On Canonical Surfaces of General Type with $K^2 = 3\chi - 10$

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1. Introduction

Let X be a smooth minimal complex surface such that $|K_X|$ has no base points and the associated map ϕ_K: $X \to \Sigma \subseteq \mathbb{P}^{p_g - 1}$ is birational. In this article I will present a classification of such surfaces under the additional assumption that $K^2 = 3\chi - 10$, and also discuss in some detail the case $K^2 = 14$, $\chi = 8$, which yields an interesting example. This classification was essentially known to Castelnuovo [C], which is not surprising since the main ingredient of the argument is an analysis of curves which are "extremal" in the sense of having the largest possible genus for their degree, and this genus bound itself was proved by Castelnuovo.

If $K^2 \leqq 3\chi - 11$, then since $\chi \leqq 1 + p_g$, we have $K^2 \leqq 3p_g - 8$. Beauville has shown [B, Theorem 5.5] that in this case ϕ_K is birationally a double covering onto a ruled surface. Moreover if $K^2 = 3\chi - 10$ and $q \neq 0$, then $\chi \leqq p_g$, so that $K^2 \leqq 3p_g - 10$, also forcing ϕ_K to have degree two. Hence with our assumptions $q = 0$ and we are considering regular surfaces at the limit of Beauville's theorem: $\chi = 1 + p_g$ and $K^2 = 3p_g - 7$, with $p_g \geqq 4$.

From the point of view of Beauville's theorem one might regard these surfaces as having minimal K^2 for their p_g. Harris, in [H], has taken the opposite tack and considered varieties with maximum p_g for their degree (which in our case is K^2), which he calls "Castelnuovo varieties". He extends Castelnuovo's arguments to varieties of any dimension, and my brief treatment here for surfaces does not differ essentially from his. Although his article is basically correct,

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