Werk

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Jahr: 1963

PURL: https://resolver.sub.uni-goettingen.de/purl?266833020_0082|log22

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Math. Zeitschr. 82, 104–105 (1963)

On finite groups with p-nilpotent subgroups

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Let p denote a prime number. As usual a finite group will be called *p-nilpotent* if and only if it contains a normal p-Sylow-complement. A non-nilpotent group all of whose proper subgroups are nilpotent will be called an *SRI-group*. SRI-groups have been very fully described by RÉDEI [4].

The aim of this note is to prove the following generalization of the Schmidt-Iwasawa Theorem:

Theorem. *A finite group all of whose proper two-generator subgroups are p-nilpotent is p-nilpotent or an SRI-group.*

Proof. Let G be a finite non-p-nilpotent group all of whose proper two-generator subgroups are p-nilpotent. Since in a p-nilpotent group the product of two elements whose orders are prime to p has order prime to p, every proper subgroup of G is p-nilpotent. Since G is not p-nilpotent, there is in G (by Theorem 14.4.7 of [2]) a p-subgroup S and an element x of q-power order (q a prime different from p) which normalizes but does not centralize S. Let T be the subgroup of G generated by S and x; then S is normal in T. If T were a proper subgroup of G, it would be p-nilpotent and so S, being its Sylow p-subgroup, would be normally complemented by the Sylow q-subgroup and then x would centralize S; thus $T=G$. Let M be a proper subgroup of G; then M has a normal Sylow p-subgroup $M\cap S$ and, since M is p-nilpotent, $M\cap S$ has a normal complement which is a q-group; therefore M is nilpotent as required.

It seems worthwhile to record two consequences of this theorem which generalize Satz 22 of [3] and Theorem 6 of [1] respectively.

A finite group all of whose proper two-generator subgroups are supersoluble has an ordered Sylow tower or is an SRI-group.

Proof. Let G be a finite non-SRI-group all of whose proper two-generator subgroups are supersoluble; then every proper two-generator subgroup of G is p-nilpotent for the smallest prime divisor p of the order of G. It follows from the theorem that G is p-nilpotent. By order induction the p-Sylow-complement N in G has an ordered Sylow tower or is an SRI-group; in the latter case N has two generators and so is supersoluble and, a fortiori, has an ordered Sylow tower. The result follows at once.

* This work was done at the third Summer Research Institute of the Australian Mathematical Society.

Hence, since a group with cyclic commutator subgroup is supersoluble:

A finite group all of whose proper two-generator subgroups have cyclic commutator subgroup has an ordered Sylow tower or is an SRI-group.

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(Received February 16, 1963)