Werk

Titel: Wahrscheinlichkeitsrechnung, Statistik und Anwendungen

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Kontakt/Contact

Digizeitschriften e.V.
SUB Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen

info@digizeitschriften.de

equations $$y_i' = \sum_j a_{ij} y_j \qquad i, j = 1, 2, \ldots, n$$

where the a_{ij} are analytic functions of one complex variable. Application is also made to the solution of the matrix equation

$$Y^{(n)} + A_1 Y^{(n-1)} + \cdots + A_{n-1} Y' + A_n Y = 0$$

where the A_i are given matrices of complex constants. *MacDuffee* (Madison).

Wahrscheinlichkeitsrechnung, Statistik und Anwendungen.

Wahrscheinlichkeitsrechnung, mathematische Statistik:

Raikov, D.: On a connection between the central limit-law of the theory of probability and the law of great numbers. Bull. Acad. Sci. URSS, Sér. Math. Nr **3**, 323—336 u. engl. Zusammenfassung 337—338 (1938) [Russisch].

The set of independent chance variables $\{x_k\}$ is said to obey the central limit law if there are sequences of numbers $\{a_n\}$, $\{b_n\}$ $(a_n > 0)$, such that the probability distribution of $\frac{1}{a_n}\sum_1^n x_k - b_n$ approaches the Gaussian distribution, and that $\frac{x_k}{a_n} - \frac{b_n}{n}$ converge $(n \to \infty)$ in probability to 0, uniformly in $k \leqq n$. The set of independent non-negative chance variables $\{y_k\}$ is said to be relatively stable if there is a sequence $\{A_n\}$ of positive numbers such that $\frac{1}{A_n}\sum_1^n y_k \to 1$ in probability and that y_k/A_n converges $(n \to \infty)$ in probability to 0, uniformly in $k \leqq n$. The author proves that if the x_k have median 0, a N.S. condition that they obey the central limit law is that the y_k with $y_k = x_k^2$ be relatively stable. The above definitions are slightly different if the expectations of the variables x_k^2, y_k exist, but the result is true for this case also. A direct proof is given, and also one using known N.S. conditions that variables obey the central limit law (Feller, this Zbl. **12**, 361; **15**, 360; Lindeberg, Math. Z. **15**, 211—225) or be relatively stable (Bobroff, C. R. Acad. Sci. URSS **15**, 239—240, this Zbl. **16**, 410; Recueil Math. Moscou [to appear in 1938]). *J. L. Doob* (Urbana).

Khintchine, A.: Theorie der abklingenden Spontaneffekte. Bull. Acad. Sci. URSS, Sér. Math. Nr **3**, 313—321 u. deutsch. Zusammenfassung 322 (1938) [Russisch].

The author treats chance events occurring at times $\{t_r\}$ (distributed at random over the t-axis in accordance with the Poisson distribution). At each such time t_r, a number x_r is determined — itself a chance variable, with a given distribution (confined to a finite interval). The distribution of the chance variable $\theta(t) = \sum_r f(x_r, t - t_r)$ is considered, continuing and making precise earlier work of Rowland (this Zbl. **15**, 261). Here the function $f(x, t)$ is a real-valued function vanishing for $t < 0$, $f(x, 0) \equiv x$, and satisfies certain regularity hypotheses. The chance variable $\theta(t)$ has a physical interpretation as the sum of damped spontaneous effects. It is shown that $U(T) = \frac{1}{T}\int_0^T \theta(t)\, dt$, the "mean effect", converges in probability to a certain constant number, and even converges with probability 1 to this constant. It is then shown, under very wide further hypotheses, that the distribution of $U(t)$ approaches the Gaussian distribution, as $t \to \infty$. *J. L. Doob* (Urbana).

Carroll, J. A.: An application of Fourier transforms to the evaluation of a class of repeated integrals arising in calculations of probability. Quart. J. Math., Oxford Ser. **9**, 176—181 (1938).

Die Arbeit bringt die vermeintlich neue Bemerkung, daß der Faltung zweier Verteilungsfunktionen die Multiplikation der zugehörigen charakteristischen Funktionen

entspricht (Laplace, 1812). Ohne Zitierung des bekannten Beweises von P. Lévy wird der zentrale Grenzwertsatz der Wahrscheinlichkeitsrechnung angeblich bewiesen, jedoch ohne Angabe der Gültigkeitsbedingungen. Demgemäß ergibt sich auch eine Verschärfung der klassischen Liapounoffschen Restabschätzung, doch ist sie nach bekannten Sätzen leicht als falsch erkennbar (vgl. etwa Cramér, Random Variables and Distribution Functions. Cambridge 1937. Kap. 7). *W. Feller* (Stockholm).

Visser, C.: The law of nought-or-one in the theory of probability. Studia Math. 7, 143—149 (1938).

Es wird bemerkt, daß man die sog. Null- oder Eins-Gesetze erklären kann, indem man sie in einem unendlichen Produktraum deutet; es handelt sich dann gewöhnlich um „homogene Mengen", d. h. Mengen η, für welche mit einer beliebigen Menge γ stets $|\gamma\eta| = |\gamma| \cdot |\eta|$ gilt. So sieht man z. B. leicht: Wenn die Konvergenz einer Funktionenfolge $f_n(x_1, x_2, \ldots)$ bloß vom asymptotischen Verhalten der Punktfolge x_n abhängt, so ist die Wahrscheinlichkeit der Konvergenz von f_n gleich 0 oder 1. *Feller.*

Khintchine, A.: Zur Methode der willkürlichen Funktionen. Rec. math. Moscou, N. s. 3, 585—588 (1938).

Bei vielen eindimensionalen Mechanismen, die man zur Illustration der Poincaréschen Methode der willkürlichen Funktionen herangezogen hat, liegt folgende Situation vor. Anfangs- bzw. Endzustand, x bzw. y, sind durch reelle Zahlen charakterisiert. Dabei ist in determinierter Weise $y = y_p(x)$, wo p ein Parameter ist. Für $p \to \infty$ gelte

$$\left(z' = \frac{dz}{dx}\right) \qquad y'_p(x) \to \infty, \qquad \frac{y'_p(x+\alpha)}{y'_p(x)} \to 1 \tag{2}$$

gleichmäßig in x bei beschränktem α. Diese Voraussetzungen (wenigstens die erste) präzisieren für große p die Forderung „kleine Ursachen, große Wirkungen". Eine diesen Bedingungen genügende Transformation $y = y_p(x)$ wird normal genannt. — Die Anfangslage x ist eine zufällige Größe. Das Experiment liefert den Wert einer Funktion $\varphi(y)$ der Endlage y. Auch $\varphi(y_p(x))$ ist dann eine zufällige Größe. Ihre Verteilungsfunktion ist bei jedem p durch diejenige von x vollkommen bestimmt. Gefragt wird nun: Welche Eigenschaften muß die Funktion $\varphi(y)$ haben, damit für $p \to \infty$ die Verteilung von $\varphi(y_p(x))$ einer Grenzverteilung zustrebt, und damit diese Grenzverteilung von der Verteilung von x und auch von dem speziellen normalen Mechanismus $y_p(x)$ unabhängig ist. Verf. beantwortet diese Frage vollständig und in sehr einfacher Weise. Gleichzeitig wird die Grenzverteilung bestimmt. Für fastperiodisches $\varphi(y)$ sind die Bedingungen erfüllt. *E. Hopf* (Leipzig).

Waschakidse, D.: Über das Maximum der Abweichung des theoretischen Verteilungsgesetzes von der entsprechenden empirischen Kurve. Trav. Inst. Math. Tbilissi 4, 101—120 u. dtsch. Zusammenfassung 120—122 (1938) [Russisch].

Consider a random variable X and denote by $F(x)$ the probability of its not exceeding any value x. Denote by $x_1 \leqq x_2 \leqq \cdots \leqq x_n$ some particular values of X and define a function $F_n(x)$ by the following conditions: (I) $F_n(x)=0$ for $x < x_1$; (II) $F_n(x) = \frac{k}{n}$ for $x_k \leqq x < x_{k+1}$; and (III) $F_n(x) = 1$ for $x_n \leqq x$. Denote finally by D_n the upper bound of $|F_n(x) - F(x)|$. Suppose now that n independent observations of the value of X are to be made and, if the values to be observed are ordered in an increasing sequence and denoted by $x_1 \leqq x_2 \leqq \cdots \leqq x_n$, the corresponding function $F_n(x)$ is to be calculated. The function $F_n(x)$ will be a random one and so will be D_n defined above. Denote by $\Phi_n(\lambda)$ the probability of D_n satisfying the inequality $D_n\sqrt{n} \leqq \lambda$. The author proves that, for $2 \leqq \lambda \leqq \frac{\sqrt{n}}{10}$, it will be $\Phi_n(\lambda) \geqq 1 - 40\sqrt{n}\, e^{-2\lambda^2}$ which is a result improving that obtained previously by Glivenko [Giorn. Ist. Ital. Attuari 4, 92—99 (1933); this Zbl. 6, 174]. The above result is then used to estimate the probability of simultaneous inequalities of the type $D_n\sqrt{n} \leqq \lambda$ for all $n \geqq N$.

J. Neyman (Berkeley, California).

Marcinkiewicz, J.: Sur les fonctions indépendantes. III. Fundam. Math. **31**, 86—102 (1938).

Diese Mitteilung ist insbesondere Verteilungsfunktionen $V(x)$ gewidmet, deren charakteristische Funktionen $\varphi(t)$ Grenzfunktionen von analytischen Funktionen sind. Es wird bewiesen: Damit $\varphi(x)$ für $|x| < A$ die Randwerte einer im Rechteck $0 \leqq \Im(z) \leqq B$, $|\Re(z)| < A$ regulären Funktion darstellt ist notwendig und hinreichend, daß $\int_{-\infty}^{0} e^{-rx} dV(x)$ für $r < A$ konvergiert. — In diesem Falle gilt eine interessante Verschärfung des Fundamentalsatzes über die Konvergenz von charakteristischen Funktionen, indem hier dafür, daß die Verteilungsfunktionen $V_n(x)$ gegen $V(x)$ streben, notwendig und hinreichend ist, daß ihre charakteristischen Funktionen $\varphi(x)$ für $|x| < A$ gegen $\varphi(x)$ streben (so daß die Gleichmäßigkeit von selbst folgt). Für jedes feste n seien nun die $x_{n,k}$ unabhängige Funktionen, $\lambda_{n,k}$ sei die Mediane von $x_{n,k}$ [d. h. $|E(x_{n,k} \geqq \lambda_{n,k})| \geqq \frac{1}{2}$ und $|E(x_{n,k} \leqq \lambda_{n,k})| \geqq \frac{1}{2}$]. Schließlich sei $\varphi(x)$ regulär für $|\Im(z)| < B$. Damit dann die Verteilungsfunktionen der Summen $\sum_n x_{n,k}$ gegen $V(x)$ streben, ist notwendig und hinreichend, daß es eine Konstantenfolge K_n gibt derart, daß für die durch

$$x^*_{n,k} = \begin{cases} x_{n,k} & \text{für} \quad |x_{n,k} - \lambda_{n,k}| \leqq K_n \\ \lambda_{n,k} & \phantom{\text{für}} \quad |x_{n,k} - \lambda_{n,k}| > K_n \end{cases} \tag{*}$$

definierten Funktionen $\lim_{n \to \infty} \sum |E(|x_{n,k} - \lambda_{n,k}| > K_n)| = 0$ ist, und die charakteristischen Funktionen der Summen $\sum x^*_{n,k}$ in jedem Bereich $|\Im(z)| \leqq r < B$ $|\Re(z)| \leqq A$ gleichmäßig gegen $\varphi(z)$ streben. [Für die Mitteilungen I. und II. vgl. dies. Zbl. **18**, 318f. Verf. berichtigt nun ein Versehen; demzufolge sind die Definitionsgleichungen von $x^*_{n,k}$ in der letzten Zeile des Ref. auf S. 318 durch die obigen Gleichungen (*) zu ersetzen.]

W. Feller (Stockholm).

Pearson, E. S.: Some aspects of the problem of randomization. Biometrika **30**, 53—64 (1938).

When testing a given statistical hypothesis, H, two kinds of errors can be committed: (1) H may be rejected when it is true and (2) we may fail to reject H when it is false. The author discusses a method devised by R. A. Fisher ("principle of randomization"), applicable to any hypothesis H, by which a number of tests may be constructed, all having the same property that the probability of committing the error of the first kind above would be reduced to any low level chosen in advance. The author shows that, if it is desired to make a choice between such tests, so as to reduce the probability of errors of the second kind, it is necessary to take into account both the specification of H and the set of admissible hypotheses, alternative to H.

J. Neyman (Berkeley, Cal.).

Pearson, E. S.: Some aspects of the problem of randomization. II. An illustration of „Student's" inquiry into the effect of „balancing" in agricultural experiments. Biometrika **30**, 159—179 (1938).

It is known that the assumptions underlying the theory of various statistical tests, which are usually applied to the results of agricultural trials, are not satisfied by the technique involved in obtaining experimental data. As a consequence the actual frequencies of cases when a correct hypothesis is unjustly rejected by the test and also the frequencies of cases when the test fails to reject a wrong hypothesis, need not be even approximately equal to the predictions of the theory. While it is desirable to develop tests based on assumptions corresponding as closely as possible to the actual technique of experimentation, it is useful to study the consequences of the application of the usual tests. The author studies the probabilities of the tests based on the "normal theory" detecting the existing differences between the agricultural treatments when (a) the distribution of these treatments among the experimental plots is random and (b) when it is balanced. The results obtained agree on the whole

with the guess of "Student": Whenever the differentiation between the treatments compared is small, the detective power of the randomized experiments is greater; otherwise the balanced experiments prove to be more efficient. *J. Neyman.*

Dwyer, Paul S.: On combined expansions of products of symmetric power sums and of sums of symmetric power products with applications to sampling. Pt. II. The fundamentals of sampling. Ann. math. Statist. **9**, 97—132 (1938).

The author now shows how the material developed in Part I (see this Zbl. **18**, 415) can be applied to actual sampling problems. The first section of the paper is devoted to a discussion of previous work on sampling, particularly as it is related to the methods here used. It is shown how these methods, especially the use of the author's double expansion theorem and of Carver functions, are adapted to different replacement laws in sampling from finite or infinite populations. A good many illustrative examples are worked out and results in tabular form up to weight 6 are given but the author is here more concerned with demonstrating the generality and utility of his methods than with giving an extensive body of new sampling results. *C. C. Craig.*

Hotelling, Harold, and Lester R. Frankel: The transformation of statistics to simplify their distribution. Ann. math. Statist. **9**, 87—96 (1938).

In testing significance informed statisticians now consider exact probabilities associated with the distribution of the statistic in question rather than treating it as though normally distributed. However the awkwardness of two and three-parameter tables is often discouraging. The authors obtain the uniform asymptotic expansion (with respect to n, the number of items in the sample) for x in terms of t, where x is the normalized value of t given by $\int_0^x \Phi(z)\,dz = \int_0^t \Phi_n(z)\,dz$. Similarly asymptotic series are given for Chi and Chi-square in terms of the generalized Student ratio, T. Short numerical tables are provided for comparison at ten, five and one percent significance levels. *Albert A. Bennett* (Providence).

Neyman, J.: Tests of statistical hypotheses which are unbiassed in the limit. Ann. math. Statist. **9**, 69—86 (1938).

The idea of a test of a statistical hypothesis unbiassed in the limit is due to the author; in a recent paper (see this Zbl. **18**, 34) he studied such a test. The present paper is devoted to such tests in general as related to simple statistical hypotheses. If w_n or $\overline{w}_n$ denote a region in the sample space into which the sample point corresponding to a sample of n may fall, a sequence of regions, $\overline{w}_1, \overline{w}_2, \ldots, \overline{w}_n, \ldots$ is said to determine a test of simple hypothesis unbiassed in the limit of the level of significance α if, on the assumption of the truth of the hypothesis, that is, that the population parameter ϑ has a given value, $P(E_n \in \overline{w}_n)$ being the probability that the sample point falls into $\overline{w}_n$, $\frac{d^2 P(E \in \overline{w}_n)}{d\vartheta^2} \geqq \frac{d^2 P(E \in w_n)}{d\vartheta^2}$, in which w_n is any region such that $P(E \in \overline{w}_n) = P(E \in w_n)$ and $\frac{dP(E \in \overline{w}_n)}{d\vartheta} = \frac{dP(E \in w_n)}{d\vartheta}$, and if $\lim_{n\to\infty} P(E \in \overline{w}_n) = \alpha$ and $\lim_{n\to\infty} \frac{dP(E \in w_n)}{d\vartheta} = 0$. By an extension of the type of argument used in the paper cited above the region $\overline{w}_n$ is determined under suitable restrictions. The author is able to discuss the power function of such a test in the limit. Finally two illustrative examples are exhibited in detail. *C. C. Craig* (Ann Arbor).

Daniels, H. E.: The effect of departures from ideal conditions other than non-normality on the t and z tests of significance. Proc. Cambridge Philos. Soc. **34**, 321—328 (1938).

The author undertakes a critical mathematical analysis of the effects due to systematic departures from independence or again from equal weight for sample observations falling individually under the normal law. The distribution of t, the ratio of sample mean to its apparent standard error calculated from the sample, and of z,

derived from the ratio of variance between groups to that within groups, are found to be insensitive to first-order systematic variations. The extent to which the approximations may be expected to hold in practice calls for further examination.

Albert A. Bennett (Providence).

Kozakiewicz, W.: Sur les conditions nécessaires et suffisantes pour la convergence stochastique. Fundam. Math. **31**, 160—178 (1938).

Die X_n und X seien stochastische Veränderliche und es werde $F(x) = P(X < x)$, $F_n(x) = P(X_n < x)$, $F_n(x, y) = P(X_n < x, X < y)$, $F_{n,m}(x, y) = P(X_n < x, X_m < y)$ gesetzt. Die entsprechenden charakteristischen Funktionen seien $\varphi(t), \ldots, \varphi_{n,m}(s, t)$. Verf. beweist, daß die folgenden vier Bedingungen einzeln notwendig und hinreichend sind dafür, daß X_n „nach Wahrscheinlichkeit" gegen X konvergiert. I. Das gleichzeitige Bestehen der Beziehungen $F_n(z, z) \to F(z)$ und $F_n(z) \to F(z)$ in jedem Stetigkeitspunkt von $F(x)$. II. Daß in gleicher Weise $\lim F_{n,m}(z, z) = F(z)$ ist, für $n \to \infty$, $m \to \infty$. III. In jedem beschränkten (s, t)-Bereiche soll gleichmäßig $\varphi_n(s, t) \to \varphi(s + t)$ streben IV. In gleicher Weise soll $\varphi_{n,n}(s, t)$ gegen eine Funktion streben [die dann von selbst $= \varphi(s + t)$ ist]. — Übertragung auf mehrdimensionale stochastische Veränderliche.

W. Feller (Stockholm).

Olds, E. G.: Distributions of sums of squares of rank differences for small numbers of individuals. Ann. math. Statist. **9**, 133—148 (1938).

The author contributes a solution to a line of investigation proposed by Hotelling and Pabst (see this Zbl. **14**, 29). He examines the distribution of the Pearsonian coefficient of rank correlation for moderate values of n. After discussing the exact problem, the author investigates approximations particularly by Pearsonian curves of Type II. Numerical tables are offered for N through 10, and for the mean value of the sum of squares through 166, with a condensed significance table for N through 30.

Albert A. Bennett (Providence).

Oboukhoff, A. M.: Sur la corrélation normale des vecteurs. Bull. Acad. Sci. URSS, Sér. Math. Nr **3**, 339—368 u. franz. Zusammenfassung 369—370 (1938) [Russisch].

The author studies problems of regression, correlation, etc., using vector, matrix, and tensor methods systematically. Thus the expectation of a chance vector is a vector, dispersion of a chance vector is a tensor, etc. The problem of the relations between two chance vectors in normal correlation (i.e. between two sets of normally correlated chance variables) is studied in detail; cf. also Hotelling, this Zbl. **15**, 407.

J. L. Doob (Urbana).

De Lury, D. B.: Note on correlations. Ann. math. Statist. **9**, 149—151 (1938).

If we are given N pairs of observations, (x_i, y_i), $i = 1, 2, \ldots, N$ in which it is known that x and y have equal population variances but different population means, the quotient, u, of the central product moment by the average of the variances is proposed as a means of estimating the population coefficient of correlation ϱ. Using results due to Fisher, the distribution of u is found in samples from normal. If $\varrho = 0$, the distribution of u is identical with that of r in samples of $N + 1$ pairs of observations. If $\varrho \neq 0$, the transformation $u = \tanh Z$, due to Fisher, is as useful as in the usual case. The author finds the variance of Z in this case and shows that $1/N - 2$ is a satisfactory approximation to it. It is remarked that u may be used in all cases in which the intraclass coefficient is appropriate and the test of significance is simpler with a negligible loss of precision.

C. C. Craig (Ann Arbor).

Kendall, M. G.: The conditions under which Sheppard's corrections are valid. J. Roy. Statist. Soc., N. s. **101**, 592—605 (1938).

A sufficient condition for Sheppard's corrections (of successively higher orders) to be effective, assuming that the absolute moments of orders involved exist is found when the range is finite and the distribution has terminal contact of order k at each end-point assuming that the $(k + 1)$st derivative of the row moment integrand is not