are not independent from one another in reality. Starting again from the original non-linear partial differential equation a new non-linear equation is derived by a simple transformation and by again appropriately applying Legendre's transformation a new linear partial differential equation is found, which readily yields a progressive wave solution. Taking a different dependent variable leads to a simple form, due to Lagrange. Introducing a Kirchhoff stream function and subjecting it to a Legendre transformation yields either a previous form or, with a new dependent variable, the well known linear partial differential equation of one-dimensional wave motion. The author then discusses the formation of singularities or discontinuities of the motion starting from a state of rest by the accelerated motion of a piston. These singularities may be found by using three dimensional representation of the solution. Next the author turns to a steady flow in two dimensions. A stream function, due to Neumann is introduced and called a stress function. Partial differential equations of the first order are derived for the differential quotients of the stress function with respect to x and to y. Then differential equations are deduced in terms of the velocity potential, due to Lord Rayleigh and to others. Solving these, equations for the stream lines are arrived at. A diagram is given, facilitating the study of these types of flow. Extensive bibliography. *M. J. O. Strutt* (Eindhoven).

Relativitätstheorie.

● **Eddington, Arthur: Relativity theory of protons and electrons.** Cambridge: Univ. press 1936. VI, 336 pag. bound 21/-.

The present book develops the author's unification-theory of relativity and quantum-physics. The author intends to give a purely deductive theory, "based on epistomological principles and not on physical hypotheses". [The word "deductive" however must not be understood in a too rigorous sense, as theorems are sometimes "proved" by showing that some characteristic alternative must be rejected.] — The book consists of two parts: "Wave-tensor-calculus" (p. 13—176) and "Physical applications" (p. 179—329). In the first one the author introduces the theory of a sedenion-system (which is called by him a "complete set of E-numbers"), i.e. a hypercomplex system with 16 principal units E_μ ($\mu = 1, \ldots, 16$), obtained as products of a "tetrad", i.e. a set of four anticommuting square roots of -1 (viz. $E_1, \ldots, E_4$). [It would have been of great use for many readers, if the author had first considered ordinary quaternion-systems, which can be obtained in the same way from two anticommuting square roots of -1; now a quaternion-system when it appears [e. g. p. 47] is called an "incomplete minor set".] — An arbitrary sedenion ("complete space vector") $T = \sum_1^{16} t_\mu E_\mu$ has 16 components t_μ and can be represented (a) by a vector in Euclidean R_{16}, (b) by a scalar $t_{(16)}$, a vector t_i ($i = 1, \ldots, 5$) and a bivector t_{ij} in R_5, (c) by a scalar, a vector, a bivector, a trivector (or vectordensity) and a quadrivector (or scalar density) in R_4. Transformations of the form $T' = e^{\frac{1}{2}E_\mu\theta_\mu} T e^{-\frac{1}{2}E_\mu\theta_\mu}$ are considered as "rotations" in R_{16}. If the sedenions are represented by (fourfold) matrices, 10 of the 16 E_μ (called "space-like") correspond with imaginary symmetrical matrices, the 6 other ones (called "time-like") with real antisymmetrical matrices. Each tetrad contains one time-like matrix and three space-like matrices. Spinvectors are always considered as the factors of "pure" or "factorizable" sedenions (i. e. idempotent sedenions with "quarter-spur" = scalar part = 1). — A covariant spinor ("wave-tensor") S of rank two is called a "strainvector". It has also 16 components s_μ, transforms according to $S' = qS\bar{q}$, where $\bar{q}$ is the transposed of q, and can be obtained from a "space-vector" T by multiplication with iE_4E_5, where E_4, E_5 are time-like and anti-commuting. [It seems that the author does not know J. A. Schouten's papers "Dirac equations in general relativity" (see this Zbl. **4**, 230), "Zur generellen Feldtheorie. Raumzeit und Spinraum" (see this Zbl. **6**, 376), where these questions have been treated in an invariant way. Cf. also W. Pauli, Über die Formulierung der Naturgesetze mit fünf homogenen Koordinaten. Teil II (see this Zbl. 8, 37), in particular Satz 3. Also van der Waerden's original paper on Spinors and Veblen's recent work on this subject are not mentionned.] — Under the group, generated by the sixteen infinitesimal rotations $e^{\frac{1}{2}E_\mu d\theta_\mu}$ with real $d\theta_\mu$, each strain-vector (after being mapped on a space-vector) moves along a ten-dimensional closed manifold (the "phase-space") in the R_{16} of all space-vectors. If S is a strain-vector, $d\omega_e$ the volume-element of the nine-dimensional space V_9, obtained from the phase-space by integrating over the scalar phase, and Σ the quotient of $S d\omega_e$ by the volume of V_9, Σ can serve three purposes: (1) its algebraic phase indicates the time, which is the argument of the complex number $\Delta = (\det \Sigma)^{\frac{1}{4}}$; (2) its symbolic phases describe a particular state ("configuration"), specified by the matrix Σ/Δ; (3) the probability of the system having this configuration to within a range $d\omega_e$ is the modulus $|\Delta|$. — From two sedenion-systems E_μ, F_μ, a system of bi-sedenions ("double wave-vectors") can be formed, generated by the 256 products $E_\lambda F_\mu$ (which are supposed to commute). In the same way as before a phase-space can be singled out, which in this case has $\frac{1}{2} \times 16 \times 17 = 136$ dimensions. — The fundamental physical assumption upon which the theory is based is: The probability-distribution of an elementary charged particle can be described by a pure wave-vector, or by the corresponding phase-vector J. Under representation in R_5 the j_i $(i = 1, \ldots, 5)$ determine the position of the particle; it can move only over an S_4 in R_5, the reciprocal radius of which is identified with the mass of the particle: $m = j_{(16)}$. If j_i is taken orthogonal to S_4, $j_5 = 0$ and j_{i5} $(i = 1, \ldots, 4)$ determines the velocity [or rather the momentum and energy] of the particle, whereas j_{ik} is the alternating product of j_i and j_{k5}. If the scalar part m is $\neq 0$, J can be brought into the form $J = im\,(E_1 + E_{23} + {}_{45} + E_{(16)})$; E_1 and E_{23} determine the directions and signs of the mechanical and magnetic spin respectively; the sign of m determines the sign of the charge. A neutral particle has only one (scalar) phase, because its probability-distribution must be invariant under reversal of the signs of charge and spin; this leads to $J = im\,E_{(16)}$. — The composition of two systems Σ, Σ' into a single one Σ_0 is performed in the following way. Let S be the strain-vector, $d\omega$, Ω, R and n the volume-element, the total volume, the radius of curvature and the number of dimensions of the phase-space and $m = \varkappa/R$ the mass of Σ; S', S_0 etc. have the same meaning for Σ' and Σ_0. Then it is assumed: (a) the law of multiplication of probabilities can be expressed by $S_0 = SS'$; (b) the time-coordinates coincide. The first assumption leads to $d\omega_0/\Omega_0 = d\omega/\Omega \cdot d\omega'/\Omega'$, or, if locally stereographic projection is used, to

$$\left(1 + \frac{r_0^2}{4R_0^2}\right)^{-n_0} = \left(1 + \frac{r^2}{4R^2}\right)^{-n} \left(1 + \frac{r'^2}{4R'^2}\right)^{-n'}. \tag{A}$$

Assumption (b) leads to $R_0^2 = RR'$ and $r_0 = r = r'$ for time-like displacements. Expansion of (A) for small r gives the "fundamental quadratic equation";

$$nm^2 - n_0 mm' + n'm'^2 = 0. \tag{B}$$

According to the author measurement of the mass of an elementary charged particle (hence $n = 10$) means combining it (hence $n_0 = 136$) with a standardized "comparison fluid" Σ', which is neutral (hence $n' = 1$). The two solutions of (B) are then supposed to correspond with the masses of the proton and the electron; the ratio of the roots is 1847,6 in well accordance with the so-called "magnetic" mass-ratio. (The "spectroscopic" mass-ratio is according to the author 136/137 times the "magnetic" value.) Consideration of the change of the mass in a static electric field leads the author to the conclusion that the two particles have equal and opposite charges. [It seems however to the ref. that the sign and the value of the charge, which also can be calculated by this method, are in discordance with experience.] — The reciprocal fine-structure-constant $hc/2\pi e^2$ is argued to be 137 by assuming that the macroscopic interaction-law between charged particles, viz. the Coulomb-law, is a consequence of the exclusion-principle (the Fermi-Dirac-law). [The original treatment in Proc. Roy. Soc. **126**, 696—728 (1930) seemed more convincing to the ref. than the present one, in particular because of a factor 2 which the author introduces into Dirac's equations and which cannot be right.] — The cosmical constants are found from the very remarkable and suggestive assumption that an Einstein-universe must be considered as a quantum-theoretical system in the ground state. It follows that the usually considered zero-level of energy corresponds with the lowest unoccupied level of the universe, and that Dirac's infinity of negative energy-levels is replaced by a finite number ($N \sim 3 \cdot 10^{79}$) of levels below this threshold-energy. By means of this assumption the gravitation-constant $\varkappa$ and the cosmical constant λ can be expressed by h, c, N and the masses of proton and electron. For N, the total number of protons and electrons in the universe, or rather the number of independent wave-functions, the author argues that the number

$$N = 2 \cdot 136 \cdot 2^{256} = n(n+1)2^{n^2}, \quad n = 2^k, k = 4,$$

must be taken. — [In § 4,7 the author believes to prove that a spinvector must necessarily be double-valued. This is erroneous: it is only proved that an individual spinvector-field in space-time cannot be invariant under arbitrary Lorentz-transformations. But this is irrelevant, as only quadratic expressions in ψ need be invariant; moreover this fact is well-known and lies at the bottom of van der Waerden's Spinor-analysis. In connection with this the author replaces h in Dirac's equation by $h/2$. But this would lead to gross contradictions with experience. Because of a slight miscalculation in § 9,4 $(U_1 - iE_4)^2$ and $(\mu - 1)^2$ have to be replaced by U_1^2 and μ^2 respectively, etc. — Although the book contains many statements which are obscure and some which are wrong, the ref. believes it to be of very great interest because of innumerable extremely profound remarks on the foundations of physic which could not be mentionned above.] *D. van Dantzig* (Delft).

Eddington, Arthur Stanley: The cosmical constant and the recession of the nebulae. Amer. J. Math. **59**, 1—8 (1937).

In this lecture Eddington describes some of the main ideas developed in his book "Relativity theory of Protons and Electrons", 1936 (see the prec. rev.). *McCrea.*

Haas, Arthur: The size of the universe and the fundamental constants of physics. Science **84**, 578—579 (1936).

Einige elementare Bemerkungen über die kosmologischen Konstanten auf Grund früherer Untersuchungen des Verf. Beispielsweise wird hervorgehoben, daß die Beziehung $M \approx \frac{10^{42}}{\sqrt{\varrho}}$ zwischen Gesamtmasse M und Massendichte ϱ des Weltalls elementar herzuleiten ist aus der Forderung, daß die Ruhenergie Mc^2 übereinstimmen soll mit dem Betrage der negativen Gravitationsenergie; rechnet man roh mit einer Euklidischen Kugel, also $M = \frac{4R}{3} R^3 \varrho$, so ergibt diese Forderung $\frac{3fM^2}{5R} = Mc^2$, wo f die Newtonsche Gravitationskonstante ist. *P. Jordan* (Rostock).

Whitrow, G. J.: World-structure and the sample principle. II. Z. Astrophys. **13**, 113 bis 125 (1937).

Einige Resultate der Milneschen Kosmologie werden unter erweiterten Voraussetzungen abgeleitet. Namentlich wird gezeigt, daß das kosmologische Prinzip ersetzbar ist durch die Forderung, daß alle Fundamentalbeobachter die gleiche Weltansicht haben in ihrer unmittelbaren („unendlich kleinen") Nachbarschaft. (I. vgl. dies. Zbl. **14**, 87.) *Heckmann* (Göttingen).

Hély, Jean: Sur une théorie synthétique de la gravitation et de l'électromagnétisme. C. R. Acad. Sci., Paris **204**, 169—170 (1937).

Ergänzende Bemerkungen zu einer früheren Arbeit des Verf. [C. R. Acad. Sci., Paris **202**, 1659 (1936); dies. Zbl. **14**, 87]. *V. Fock* (Leningrad).

Yano, Kentaro: La théorie unitaire des champs proposée par M. Vranceanu. C. R. Acad. Sci., Paris **204**, 332—334 (1937).

Vranceanu hat, ausgehend von einer nichtholonomen V_5^4 $(ds^5 = 0)$ in einer V_5 mit Linienelement

$$d\sigma^2 = G_{\lambda\mu}\, dx^\lambda\, dx^\mu = -(ds^1)^2 - (ds^2)^2 - (ds^3)^2 + (ds^4)^2 + (ds^5)^2 \tag{1}$$

(die $ds^\varkappa$ sind Pfaffsche Formen) eine generelle Feldtheorie konstruiert (vgl. dies. Zbl. **15**, 279). Er nimmt an, daß in bezug auf die verwendeten Bezugssysteme gilt: 1. die Koeffizienten in $ds^\varkappa$ und daher die $G_{\lambda\mu}$ sind von x^5 unabhängig; 2. ds^h $(h = 1, \ldots, 4)$ ist von dx^5 unabhängig; 3. $G_{55} = 1$. Diese drei Annahmen werden in dieser Arbeit präzisiert. Verf. geht aus von einer V_5, welche eine infinitesimale Bewegung $'x^\varkappa = x^\varkappa + v^\varkappa \delta t$ gestattet, und es wird angenommen, daß die Bewegung alle Punkte über denselben Abstand verschiebt $(v^\varkappa v_\varkappa = \text{Konstante})$. Wählt man nun das Koordinatensystem derart, daß $v^\varkappa$ die Bestimmungszahlen $(0, 0, 0, 0, 1)$ hat, so zeigt sich, daß $G_{\lambda\mu}$ von x^5 unabhängig ist, $G_{55} = 1$ ist und $d\sigma^2$ sich in der Form (1) schreiben läßt. *J. Haantjes* (Delft).

Bhabha, H. J.: The wave equation in conformal space. Proc. Cambridge Philos. Soc. **32**, 622—631 (1936).

Verf. schließt sich an eine Arbeit von Dirac an [Ann. of Math. **37**, 429 (1936); dies. Zbl. **14**, 80], wo der projektive fünfdimensionale Raum mit sechs homogenen Koordinaten x_μ betrachtet wird, die der Relation $x_\mu x_\mu = 0$ genügen. Es wird das Vektorpotential A_μ (homogen in x_ν vom Grade -1) eingeführt, das den Bedingungen $\frac{\partial}{\partial x_\nu}(x_\mu A_\mu) = 0$ unterworfen wird. Mit dessen Hilfe werden die Maxwellschen Gleichungen im projektiven Raum geschrieben und deren Beziehungen zu den Gleichungen im euklidischen Raum untersucht. Zum Schluß wird die 6dimensionale Spin-Wellengleichung geschrieben; im euklidischen Raum führt dieselbe auf eine entartete (mit $1 + \alpha_5$ multiplizierte) Diracsche Gleichung. *V. Fock* (Leningrad).