Werk

Titel: Fourierreihen:

Jahr: 1931

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Behauptung folgt. Der 2. Satz von Ramanujan behauptet die beiden Kettenbruchentwicklungen

$$4\int_0^\infty \frac{xe^{-x\sqrt{5}}}{\mathfrak{Cof}\, x}\,dx = \frac{1|}{|1} + \frac{1^2|}{|1} + \frac{1^2|}{|1} + \frac{2^2|}{|1} + \frac{2^2|}{|1} + \frac{3^2|}{|1} + \frac{3^2|}{|1} + \cdots, \tag{2}$$

$$2\int_0^\infty \frac{x^2e^{-x\sqrt{3}}}{\mathfrak{Sin}\, x}\,dx = \frac{1|}{|1} + \frac{1^3|}{|1} + \frac{1^3|}{|1} + \frac{2^3|}{|1} + \frac{2^3|}{|1} + \frac{3^3|}{|1} + \frac{3^3|}{|1} + \cdots. \tag{3}$$

Es bedeute

$$\Phi_n(x) = \int_0^\infty \frac{t^n e^{-\frac{t}{x}}}{\mathfrak{Cof}\, t}\,dt; \quad \Psi_n(x) = \int_0^\infty \frac{t^n e^{-\frac{t}{x}}}{\mathfrak{Sin}\, t}\,dt.$$

Für Φ_0, Φ_1, Ψ_1, Ψ_2 hat schon Rogers (l.c.) Kettenbruchentwicklungen gegeben. Verf. gibt dafür neue Beweise und Darstellungen für beliebiges n. Für $n = 1$ erhält man durch eine längere Rechnung

$$\Phi_1(x) = \frac{x^2|}{|1-x^2} + \frac{2^2x^2|}{|1} + \frac{2^2x^2|}{|1-x^2} + \frac{4^2x^2|}{|1} + \frac{4^2x^2|}{|1-x^2} + \cdots.$$

Da $4\Phi_1(1/\sqrt{5})$ die linke Seite von (2) ist, so ist damit (2) bewiesen. Ferner ergibt sich durch ganz ähnliche Betrachtungen

$$\Psi_2(x) = \frac{x^2|}{|1-x^2} + \frac{2x^2|}{|1} + \frac{2\cdot 1^3x^2|}{|3(1-x^2)} + \frac{2\cdot 2^3x^2|}{|1} + \frac{2\cdot 2^3x^2}{|5(1-x^2)} + \frac{2\cdot 3^3x^3}{|1} + \cdots.$$

Da $2\Psi_2(1/\sqrt{3})$ die linke Seite von (3) ist, so ist (3) damit bewiesen.

G. Wiarda (Dresden).

Verblunsky, S.: The symmetric derivative and its application to the theory of trigonometric series. Proc. Cambridge philos. Soc. **27**, 163—173 (1931).

The author proves a series of theorems on the symmetric derivative and applies them to the uniqueness of trigonometric series. We quote as examples the following theorems. (a) If the upper and lower symmetric derivatives ($\overline{D}F$ and $\underline{D}F$) of a function $F(x)$, defined for $a < x < b$, are finite and if there exists a function φ, integrable in the sense of Denjoy-Perron and such that $\overline{D}F \geqq \varphi \geqq \underline{D}F$, (*) then, for almost all x, $F(x)$ is equal to $\int_a^x \varphi(t)\,dt + C$, C denoting a constant. (b) Let, for a trigonometric series

$$\sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx) \tag{**}$$

with coefficients $O(n^2)$, the integrated series $\sum_1^\infty (a_n\sin nx - b_n\cos nx)/n$ have its upper and lower Poisson sums finite everywhere and let F be the first of them. Then, if (*) is true, with φ integrable in the sense of Denjoy-Perron, (**) is the Fourier series of φ.

A. Zygmund (Wilno).

Moore, C. N.: Types of series and types of summability. Bull. amer. math. Soc. **37**, 240—250 (1931).

General remarks on the history of divergent series and on the applications of some methods of summability to power, Dirichlet's and Fourier series. *A. Zygmund* (Wilno).

Verblunsky, S.: On summable trigonometric integrals. Proc. Cambridge philos. Soc. **27**, 174—183 (1931).

The paper contains a proof of the following theorem. Let the coefficients $A(\lambda)$, $B(\lambda)$ of a trigonometric integral

$$\int_0^\infty \{A(\lambda)\cos\lambda x + B(\lambda)\sin\lambda x\}\,d\lambda \tag{1}$$